

REVIEW OF J.F.A.K VAN BENTHEM, THE LOGIC OF TIME
(forthcoming in Journal of Symbolic Logic)

This is a philosophically and mathematically imaginative survey of topics in tense logic (where that subject is construed much more broadly than usual.) The author complains in the introduction about the low ratio of ideas to prose in many philosophy books. This work certainly succeeds in bucking the trend. In 239 pages there are eighty-one numbered theorems, scores of unnumbered results and conjectures, and scattered philosophical remarks about Kant's first and second antinomies, Zeno's paradoxes, connections between properties and comparative relations, the significance of completeness proofs, the meaning of "continuity", the nature of philosophy and diverse other matters.

Despite the wide variety of topics addressed the book is nicely structured. It is divided into two main parts--"temporal ontology" and "temporal discourse" and an appendix on space. The labeling of the parts is a little misleading, since both parts concern languages in which temporal matters can be expressed. In Part I the languages are classical predicate logics with one or two special relation symbols and in Part II they are sentential logics with one, two, or three special tense operators. Within each part topics having to do with "temporal points" (i.e., instants) are addressed first, topics having to do with "periods" second, and connections between points and periods third.

The points section of Part I can be viewed as a treatment of the model theory of strict partial orders. The investigation proceeds from relatively meager logical resources to more powerful resources. We learn first, for example, that for each n the theory of strict linear orders of n elements can be expressed with purely universal axioms and that no extensions of these theories can be so expressed. The theories of dense linear order and discrete linear order (requiring $\forall\exists$ and $\forall\exists\forall$ axioms, respectively) are shown to be syntactically complete by the "back and forth" argument. Both of these theories admit a number of non-standard models, i.e., models other than the rationals under $<$ and the integers under $<$. Since all these models share the same first order theories it is natural to ask whether they could be ruled out by some higher-order conditions. It is shown quite easily that for the integers this can be done by a (Π^1_1) axiom of Dedekind continuity. (If all the A points precede all the not- A points there is a "dividing" point whose predecessors are all A -points and whose successors are all not- A points.) This axiom fails in the rationals, however, and in the reals it does not rule out all the non-standard models. Several other higher order conditions are considered, including "connectivity" (for any points s and s' there is a sequence t_1, \dots, t_n such that $t_1=s$, $t_n=s'$ and for $i=1, \dots, n-1$ either $t_i < t_{i+1}$ or $t_{i+1} > t_i$); two versions of "symmetry" (every model is isomorphic to its converse; the substructure formed from the successors of a given point and that formed from its predecessors are isomorphic); two versions of "reflection" (every open interval is isomorphic to the whole; the original structure is isomorphic to the result of replacing each point by a copy of the original structure); and two versions of homogeneity (for any pair of points t and t' there is an automorphism that maps t to t' ; for any two pairs t, t' and s, s' there is an automorphism that maps t to t' and s to s'). There is some discussion of connections among these conditions and of some first order consequences of them, but the question of whether they are enough to characterize completely any natural structures remains largely unanswered. Perhaps the best result along these lines is that there are exactly three countable structures satisfying first order conditions for strict linear orders and the second version of homogeneity mentioned above: the rationals, the integers, and the structure

formed by replacing each rational by a copy of the integers.

In the period section of Part I the language considered contains a predicate of inclusion (\subseteq) as well as precedence ($<$). Natural models here are the open intervals of rationals with set-theoretic inclusion and strict (non-overlapping) precedence and the closed intervals of integers with the same relations. The first order theory of the rationals structure is axiomatized by seventeen axioms. (Completeness of the axioms is established by proving the stronger result that the theory is countably categorical.) Most of the axioms are short and easy to grasp, but one of them occupies more than five lines. If the long axiom is omitted and the axiom of density is replaced by a second-order axiom of foundation a categorical characterization of the integers structure is obtained. It is suggested that a first order axiomatization can be obtained by replacing foundation with "atomicity" ($\forall x \exists y \subseteq x \forall z \subseteq y (z = y)$). But the details of the proof are apparently too messy to be included.

Periods, of course, can be regarded as "stretches" of points. In the "connections" section the author shows that a surprising number of the conditions used to characterize the natural period structures hold for all structures that are obtained strict partial orders by identifying periods as intervals of points. He had argued earlier, however, for the significance of period structures satisfying much weaker conditions and he shows that these can be generated from point structures if the set of periods is allowed to be an arbitrary collection of sets of points that is closed under non-empty intersections. (A period is apparently to be thought of as the time occupied by an event. It might well have gaps.) A reduction in the other direction is more problematic. If every period contains atoms (periods including no other periods) then it is natural to take these as the points. If not, it is suggested that the points can be taken as the set of all filters, or alternatively as the set of maximal filters. Either identification would entail that there are more points than periods, a result that seems somewhat counterintuitive. Neither identification always "commutes" with the reverse construction in the sense that the point structure constructed from periods that are themselves intervals of points is isomorphic to the original point structure. J. Burgess ("Beyond Tense Logic", *Journal of Philosophical Logic*, 13 235-248) and S.K. Thomason ("On Constructing Instants from Events",) have recently discussed a construction that is somewhat better on this score. Points are identified with "termini", which turn out to be equivalence classes of pairs of adjoining periods. But this construction is less general--Burgess and Thomason require that the underlying point structure be a dense linear order without endpoints.

In the final section of Part I, the author suggests that periods (and hence points) might be constructed out of a primitive notion of "event", thereby reversing the "classical" analysis in which events are analyzed as interval-plus-description. This suggestion prompts some engaging philosophical digressions, but we never really see how the program is to be carried out. (We do get interesting comments about a related idea of J.A. Winnie to reduce both space and time to a primitive notion of causality.) In fact the author's remark that the relation of inclusion among events can be defined from the relation of precedence suggests that he may not really be taking the priority of events seriously. For on such a definition the war between Iran and Iraq would include the Mets' victory in the 1986 World Series. This would seem to make sense only if we are already thinking of events as temporal periods.

The "point" section of Part II contains a survey of tense logic of the traditional, Priorean sort. As in Part I model theoretic concerns predominate. We learn that on finite, connected frames, verifying the same tense-logical formulas implies isomorphism, whereas on infinite, connected frames it does not. We get preservation theorems: generated submodels, p-morphic images and ultrafilter extensions. We get an extensive survey of correspondences (and the lack of them) between formulas

of tense logic and first order classical logic. Finally we get a sampling of completeness and incompleteness results.

For period structures there is no such well-developed tense logic to survey. Here the task is rather to devise a framework sufficiently broad to enable some of the initial explorations to be discussed and sufficiently simple to allow a reasonable chance for successful investigation. The author chooses simply to add to the Priorian tense logic an operator \Box with the reading "at all included periods". This choice turns out to be quite successful. Humberstone's strict negation can be expressed as $\Box\text{-}A$. Something like Cresswell's durational conjunction can be expressed by $\Diamond A \wedge \Diamond B \wedge \Box \Diamond (A \vee B)$. Various suggestions about the kinds of regularity that periods must exhibit can be formulated as restrictions on admissible valuation functions for the new logics or as axioms for them. Technically, one can ask the same kinds of correspondence and completeness questions as before. Correspondences that concern $<$ or i alone can be carried over directly from the previous discussions, but there are some interesting "mixed" cases. The axiom $\mathbf{FA} \rightarrow \Box \mathbf{FA}$, for example corresponds to "right monotonicity": $\forall \mathbf{x} \mathbf{y} (\mathbf{x} < \mathbf{y} \rightarrow \forall \mathbf{u} \subseteq \mathbf{y} \mathbf{x} < \mathbf{u})$. Completeness proofs are considerably harder than before. The problems of axiomatizing period logic for the rationals and for the reals remain open.

The constructions of points from periods and periods from points discussed in Part I yield morphisms between frames. The connections section of Part II considers how these might be extended to models. There appears to be no way to get complete correspondences, i.e., correspondences such that a model of one variety verifies a formula if and only if the associated models of the other variety do. But by placing appropriate restrictions on admissible valuations or on the kind of formulas considered some partial correspondences are obtained.

The preceding outline has treated only the main themes in the book. The many digressions and "discussions" are clearly intended to be an equally important part of the book.

The book as whole is remarkably free of philosophical dogma. The only doctrine that is consistently advocated and followed is a kind of tolerance for diverse ideas and a willingness to postpone judgement among competing research programs. This tolerance occasionally seems excessive, as when it is suggested that the additional technical difficulties of the period framework might merely be a result of "troubles in adapting to a new conceptual environment", or when the view of philosophy as "the science of lost causes" is cited with enthusiastic approval. (Apparently the postponement of judgement is to be permanent.) Nevertheless the attitude is a refreshing change from the usual one that other authors are either allies or enemies and that points should be scored against the latter group whenever possible.

The choice of technical topics and methods is never determined by mere convention. Standard Makinson-Henkin completeness proofs and decidability-by-filtration arguments are here, but they are a small portion of the book and their significance is discussed rather than assumed. Similarly, the standard notion of p-morphism is defined, but alternative kinds of maps, both stronger and weaker are also considered. When logical formalism is employed in the service of philosophy it is usually to do what it does best--to sharpen and disambiguate vague ideas (continuity, symmetry, homogeneity, for example). More often the motivation is in the reverse direction. Philosophical concerns prompt new technical investigations. One indication of this is the number of unproved conjectures, open problems and suggested areas of research. There are more than twenty. It might be worth listing a few of them here.

1. (p28) Consider the ordering $<$ on pairs of rationals such that $(u,v) < (u',v')$ if and only if $u < u'$ and $v < v'$. What is the first order theory of this relation.

2. (p31) Can the ordering of the rationals be characterized by Σ_1^1 sentences?
3. (p44) Given two strict partial orders A and B, let $A \cdot B$ be the result of replacing each element of A by a copy of B. Characterize the class of orders A for which $A \cdot A$ is isomorphic to A.
4. (p46) Can a finite strict partial order be homogeneous, but not symmetric?
5. (p160) Can every definable "purely past" condition on tense logical frames be defined without the future operator?
6. (p182) What is the priorean tense logic of homogeneous frames?
7. (p215) What is the period logic (in the sense described above) for intervals of integers?

The book is written in an informal, chatty style. Proofs are frequently sketched, rather than set forth in detail. Most occupy less than a page; the longest is under five pages. In some cases (Kamp's functional completeness of "since" and "until", Goldblatt and Thomason's characterization of the tense-logically definable formulas, Burgess's decidability of the monadic Π_1^1 theory of the integers, for example) the reader is simply referred to the relevant literature. All of this makes the book quite readable. It makes it a little less valuable as a reference, however. For a serious reader, it may be somewhat frustrating to read remarks like "Correspondence Theory tells us that.." (p 180), or "a more complex unraveling argument may be used to prove..." (p212). The same kind of sketchiness is present in the philosophical discussions. In discussing Zeno's arrow, for example, the author argues that when one takes events as one's "primary stock of data" and instants as "fictitious limits" the problem simply "evaporates". But surely there is more to be said here. Whether instants are natural or man-made, Zeno's arrow at least shows that the ordinary ideas about the relation between what happens at instants and at periods need to be reworked. I do not wish to suggest that the book would be improved by filling in missing details on all the topics addressed, but only to point out that there is a price to paid for its range and readability.

The book is, on the whole, quite well produced. There are a couple of slips that could cause misunderstanding. On page 38 line 9, the last '<' is probably supposed to be a '>'. (At any rate that version of the definition would fit better with the motivating remarks on the previous page.) On page 46 "the cube of Figure 16" should read "the octahedron of figure 17." On page 151, line 2 the first "M" should read "M*". In the statement of theorem II.3.1.4 on page 199 the last clause should read " $\phi \leftrightarrow \Box \phi$ and **NOT NOT** $\phi \leftrightarrow \phi$ are valid for the relevant valuations."

Overall this is an admirable work. I believe it will change and enrich the field.

Johan van Benthem. *The Logic of time. A model-theoretic investigation into the varieties of temporal ontology and temporal discourse*. Second edition of LII 874. Synthese library, vol. 156. Kluwer Academic Publishers, Dordrecht, Boston, and London, 1991 xxii+280 pp.

This second edition corrects "only a few errors and misprints" in the original text and adds the following: a one-page preface, three pages of "addenda et corrigenda", a fifteen page appendix entitled "Sept Ans Apres," and five pages of additional references. Even the pagination and the index of the original are kept intact. Someone who already owns the first edition might well question the wisdom of spending \$89.00 for twenty-five new pages. For someone who does not already have the first edition, however, this is still the most wide-ranging, up to date and readable survey of tense logic available.

The review of the first edition noted a willingness to examine a large number and variety of ideas and a refreshing tolerance of competing research programs. In the eight years between editions these attitudes seems to have imbued the field. The new appendix chronicles further work on the technical questions raised in the first edition and sketches some new directions that the study of tense and time has taken within the linguistics and computer science communities. The price for all this diversity and tolerance seems to be that the field lacks the focus that it once had. One does not get a sense, reading the appendix, of milestone results or outstanding open problems. Indeed the work that is being done is motivated by widely divergent considerations: the search for systems in which to represent reasoning about the behavior of computer programs, the enterprise of formalizing "common-sense" physics, the traditional semantics (i.e., contribution to sentential truth value) of tense and aspect, the cognitive science (i.e., contribution to sentential processing) of tense and aspect, and still, to some extent, the older concerns of representing arguments in which time and tense play a role and analyzing philosophical puzzles about time. The main unifying theme that the author suggests for current work is "ontological pluralism". We have learned that it is not productive to insist on the priority of instants or periods or events or any other temporal ontologies. Instead we now quite properly study all of these and investigate the relations among them. In the text of the

book, the treatments of period- and point-based views of time are largely segregated in separate chapters and the brief discussions of their relations is primarily concerned with reductions in each direction. Recently at least one author seems to have taken van Benthem's pluralism even more seriously. (Alexander Bochman's, 'Concerted Instant-Interval Temporal Semantics', parts I and II, *Notre Dame Journal of Formal Logic*, vol 31 numbers 3 and 4, 1990 considers models which encompass periods, points, and certain relations among them).

"Sept Ans Apres" is a thematic essay, and one will find few precise statements of new results and even fewer details of arguments for those results. Together with the bibliography, however, it does provide a valuable guide to the recent literature. In addition to the Bochman papers, one might wish to add the survey papers by Burgess and R. Thomason in *Handbook of Philosophical Logic*, vol II, Kluwer (1984) and van Benthem's own survey in *Handbook of Logic in Artificial Intelligence and Logic Programming* Oxford (forthcoming). Of the six open problems mentioned in the review of the first edition, only the fourth is reported as having been solved: Hendrik Lenstra has found a homogeneous finite order that is not symmetric. The first edition had relatively few typos and misprints and most of those seem to have been corrected. One of the misprints noted in the review of the first edition has not been corrected: the first occurrence of "M" on page 151 should be "M*".

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