Two-Dimensional Logics and Two-Dimensionalism in Philosophy

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INTRODUCTION

The label "two-dimensional" has been applied to disparate logical systems. The first section of Humberstone 2004 nicely conveys the diversity. For present purposes, we may understand an n-dimensional logic to be one in which sentences acquire truth values relative to n parameters. Thus classical propositional logic, which represents sentences as being true or false *simpliciter*, is zero-dimensional. Common modal or tense logics, which represent sentences as being true or false relative to a world or a time, are one-dimensional. Logics aiming to represent both tense and modality assign truth values relative to both time and world. (Thomason 1984 provides an overview.) They are two-dimensional. For other linguistic phenomena, other parameters have been employed: for personal pronouns, a speaker, an addressee and a salience order among some class of individuals; for *here* and *there*, a pair of locations; for more faithful representations of tense (in a remarkable anticipation of ideas discussed here, dating to Reichenbach 1947), a time of utterance and a time referred to. It may sometimes be appropriate to view some parameters as constructed from others. To represent a notion of *historical* necessity according to which the past is determined and the future is open, worlds may be seen as constructed out of instants and their temporal order. Proper understanding of actions, events, states and the like may require that we distinguish among sentences evaluated at instants and intervals and to view one as constructed from the other. For other applications it may be appropriate to view the parameters as primitive and perhaps independent. To represent tense and metaphysical necessity, we may take time and world as independent parameters. Alternatively, we may take time to be something internal to each world, allowing for the possibility that it has different structures in different worlds. All such generalizations of standard modal and tense logics may be regarded as many-dimensional, the dimension corresponding to the number of parameters to which the truth of a formula is relativized.

Recent interest in "two-dimensionalism" in philosophy stems from the observation that parameters of the kind mentioned above may play two distinct roles in the determination of a sentence's truth value. The nature of these roles and the parameters that are supposed to play them is delineated differently by different authors, but it may be convenient to use the label 2D for two-dimensional logics whose parameters are both possible worlds. We thereby encompass the frameworks of Davies and Humberstone 1980 and Stalnaker (1981, 2004), an approximation to that of Chalmers (2006, 2006a, forthcoming), and illustrative fragments of those of Lewis 1981 and Kaplan (1978, 1979). We label the dimensions *utterance* world and *evaluation* world, or simply (to avoid prejudicing later interpretations) *u-world* and *e-world*. From a technical perspective, much of the interest in logics of this kind stems from the fact that the homogeneity of the two parameters allows each to take the value of the other. From a philosophical perspective, the dual role of possible worlds has been said to illuminate some of the most deep and central topics in contemporary philosophy: the existence and nature of contingent *apriori* and necessary empirical truths, the role of conceptual analysis in the acquisition of knowledge, the link between conceivability and necessity and the nature of Fregean sense, secondary qualities, narrow content, meaning and communication. Whether these claims of illumination are warranted depends on which, if any, of the various interpretations of the 2D framework are coherent. This is a contested matter. In this survey we will describe the logical machinery and outline its deployment by a few philosophers, emphasizing possible connections to the contingent *apriori* and necessary empirical.

LOGICAL MACHINERY

However contested its various interpretations, the technical machinery required for a 2D logic of the kind described above is relatively straightforward. We may take a 2D *model* to be a pair M=(W,V), where W is a non-empty set (the possible worlds) and V is a function (the valuation) that assigns a subset of $W \times W$ to each sentence letter. A sentence letter p is true in M at a pair (u,v) of possible worlds (u-world and e-world, respectively) if $(u,v) \in V(p)$. We may consider languages with a variety of connectives and constants, including those with the following clauses defining A is true at (u,v) in M (written $(u,v) \vDash_M A$). (The quantifiers here are all taken to range over W.)

$(u,v) \vDash_M \Box A$	iff	$\forall u' \forall v' \ (u',v') \vDash_M A$
$(u,v) \vDash_M \boxed{1} A$	iff	$\forall v' \ (u,v') \vDash_M A$
$(u,v) \vDash_M \boxed{2} A$	iff	$\forall u' \ (u',v) \vDash_M A$
$(u,v) \vDash_M ① A$	iff	$(u,u) \vDash_M A$
$(u,v) \vDash_M ② A$	iff	$(v,v) \vDash_M A$
$(u,v) \vDash_M \otimes A$	iff	$(v,u) \vDash_M A$

$(u,v) \vDash_M \boxed{\bigcirc}A$ iff $\forall w\ (w,w) \vDash_M A$

$(u,v) \vDash_M \boxed{\cdot\,^{\cdot}}A$ iff $\forall w\ (v,w) \vDash_M A$

$(u,v) \vDash_M \boxed{\cdot}A$ iff $\forall w\ (w,u) \vDash_M A$

$(u,v) \vDash_M \mathbf{I}$ iff $u=v$

We may say that a formula A is *valid* if, for any model M and any world u of M $(u,u)\vDash_M A$, and that A is *strongly valid* if, for any M and any worlds u,v of M $(u,v)\vDash_M A$. (Motivation for the definitions of validity will be suggested in subsequent sections.)

These connectives (or their informal counterparts) have appeared in the logic and philosophy literatures on two-dimensionalism under a variety of names and notations. The nomenclature above follows a convention once suggested by Brian Chellas. Connectives roughly analogous to the standard necessity operator are represented by something whose outer boundary is a square. Duals of these connectives are named by rotating the boundary forty five degrees to to obtain a diamond. For example, $\boxed{\circ}$, the dual of $\boxed{\bigcirc}$, is a conective having the truth conditions:

$(u,v)\vDash_M \boxed{\circ}A$ iff $\exists w\ (w,w)\vDash_M A$. If a connective **C** is self dual (so that **C**A always takes the same truth value as ¬**C**¬A) then it is represented by something whose boundary (in the cases above, a circle) remains the same when so rotated. Beyond these conventions, numbers appearing within the connective indicate the coordinate on which the truth of a formula made by applying the connective depends. ⚁ and ⊡ are intended to evoke the image of dominos. If their truth clauses are written in terms of accessibility relations those relations, (t,u)R(v,w) iff u=v, and (t,u)R(v,w), iff t=w are the relations that permit dominos to be placed to the right and left of others in play. It is common to view the pairs of worlds as forming an array in which (for reasons unclear) the vertical distance from the top determines the u-world and the horizontal distance from the left indicates the v-world. On this understanding, the identity pairs (u,u) form a *diagonal* from upper left to lower right. $\boxed{1}$A, $\boxed{2}$A and ⧇A are true at a pair if A is true throughout the corresponding row, the corresponding column and the diagonal, and the other connectives can likewise be understood in geometric terms.

A logical system based on the connectives □, $\boxed{1}$, $\boxed{2}$, ①, ②, and ⊗ (and the notion of strong validity) referred to as the *basic* two-dimensional system ***B*** is axiomatized in Segerberg 1973. Systems based on ⚁ and ⚀ are investigated in Kuhn 1989 and Venema 1992.

These connectives are interdefinable in various ways. For example, each square and diamond connective is interdefinable with its dual.

□A may defined as $\boxed{1}$ $\boxed{2}$A, $\boxed{2}$ $\boxed{1}$A, ⚀⚀A, or ⚁⚁A;

⧇A, as $\boxed{2}$ ①A or $\boxed{2}$ ①A;

②A, as ⊗①A or $\boxed{:}$(**I**∧A);

①A, as ⊗②A or $\boxed{\cdot}$(**I**∧A);

$\boxed{2}$A, as ⊗$\boxed{1}$⊗A or ②$\boxed{\cdot}$**A**;

$\boxed{1}$A, as ⊗$\boxed{2}$⊗A or ①$\boxed{:}$A;

$\boxed{:}$A, as ② $\boxed{1}$A or ⊗$\boxed{1}$A;

$\boxed{\cdot}$A, as ① $\boxed{2}$A or ⊗$\boxed{2}$A.

Thus all of the above connectives except **I** are definable in ***B*** and in fact they can all be defined in a language that contains just $\boxed{1}$, **①**, and ⊗ as primitive. Adding $\boxed{2}$ and **②** to these three connectives allows us to obtain a useful normal form result. To see this, note that the following biconditionals are valid (where each i and j inside a circle or square is either 1 or 2, ĩ is the "opposite" number as i and ◯ may be **①**, **②**or ⊗:

ⓘⓙA↔ⓘA

ⓘ⊗A↔ⓘA

ⓘ$\boxed{i}$A↔$\boxed{i}$A

ⓘ$\boxed{\tilde{\imath}}$A↔$\boxed{i}$⊗A

$$\otimes\textcircled{i}A\leftrightarrow\textcircled{$\tilde{\imath}$}A$$

$$\otimes\otimes A\leftrightarrow A$$

$$\otimes\boxed{i}A\leftrightarrow\boxed{\tilde{\imath}}\otimes A$$

$$\bigcirc(A\vee B)\leftrightarrow\bigcirc A\vee\bigcirc B$$

$$\bigcirc\neg A\leftrightarrow\neg\bigcirc A$$

By means of these biconditionals, we can push every circular connective occurrence inward, past occurrences of squares and boolean connectives, shortening all strings of circular connectives to length at most one, until each occurrence of ①, ②and ⊗ governs only a sentence letter. The observation that all formulas can be written in this “inner circle” form facilitates a proof that the basic two-dimensional logic ***B*** is equivalent to the fragment, PL(x,y), of ordinary predicate logic that contains only binary predicate letters and the two variables x and y. Note that the 2D models M=(W,V) can be viewed as models for dyadic predicate logic. W becomes the *domain* of the model and the set of pairs that V assigns to the i’th sentence letter $\mathbf{p}_i$ becomes the extension of the i’th predicate, $\mathbf{P}_i$. A translation t that maps the inner circle formulas one-one onto PL(x,y) can be defined as follows.

$t(\mathbf{p}_i) = \mathbf{P}_i xy$

$t(①\mathbf{p}_i) = \mathbf{P}_i xx$

$t(②\mathbf{p}_i)=\mathbf{P}_i yy$

$t(\otimes\mathbf{p}_i)=\mathbf{P}_i yx$

$t(A\vee B) = t(A)\vee t(B)$

$t(\neg A) = \neg t(A)$

$t(\boxed{1}A)=\forall y\ t(A)$

$t(\boxed{2}A)=\forall x\ t(A)$

We can now establish by formula induction that truth of an inner-circle formula A at the pair (u,v) corresponds to truth of the predicate logic formula t(A) when u and v are assigned to x and y. In symbols, $(u,v) \vDash_{MA}$ iff $M \vDash t(A)[u,v]$ (and consequently, for all φ in PL(x,y), $M \vDash \varphi[u,v]$ iff $(u,v) \vDash_M t^{-1}(\varphi)$.) Thus, the inner circle formulas and the formulas of PL(x,y) are essentially the same and, since any formula with connectives above other than **I** is equivalent to an inner circle formula, the full two-dimensional logic without **I** is essentially PL(x,y). It is perhaps not as obvious as it may seem that every two-dimensional formula can be expressed with two variables. The formula $\boxed{.^{\cdot}}\ \boxed{.^{\cdot}}\ (p_1 \wedge \boxed{.^{\cdot}}(p_2 \wedge \boxed{.^{\cdot}}p_3))$, for example, might naturally be associated with the four-variable formula $\exists x \exists y \exists z \exists w(P_1xy \wedge P_2yz \wedge P_3zw)$. By the construction above, however, it can be seen to be equivalent to $\exists x(\exists y(P_1xy \wedge \exists x(P_2yx \wedge \exists y(P_3xy))))$. Our translation can be extended to include **I** by setting t(**I**) to (x=y) (and t(①**I**), t(②**I**), t(⊗**I**) to (x=x),(y=y) and (y=x)), thereby establishing the equivalence of the full system to the version of PL(x,y) with identity, which we might write $Pl^{=}(x,y)$. Since identity is not definable in PL(x,y), it follows that **I** is not definable in ***B***. None of the various flavors of two-dimensionalism make use of all the connectives described above, but most can be understood as built upon some of them, and hence as built upon some fragment of PL(x,y).

DAVIES AND HUMBERSTONE

The application in which the technical apparatus sketched above lies closest to the surface is contained in the influential Davies and Humberstone 1980,where it is used to explicate a distinction drawn in Evans 1979 between “deep” and “superficial” necessity. $(u,v)\vDash_M A$ is read “A is true *at* v from the perspective of u as the actual world.” $\boxed{1}$ and $\textcircled{1}$ (written there as $\Box$ and $\boldsymbol{A}$) are read *necessarily* and *actually*. In a language with just these two connectives, the truth value of A at (u,v) depends only on the truth values of its subformulas at pairs (u,v′). u “stores” the world regarded as actual so that the *actually* operator can refer to it even when that operator lies within the scope of necessity operators. While this may be a plausible characterization of the semantic roles of *actually* and *necessarily*, it has the peculiar consequence that $\textcircled{1}A\leftrightarrow\boxed{1}\textcircled{1}A$ is strongly valid. If, at v, A is true at u from the perspective of u as actual then, at any w, A is true at u from the perspective of u as actual. Davies and Humberstone diagnose the apparent peculiarity as stemming from the thought that “another world might have been actual,” and suggest $\boxed{2}$ (with the reading *fixedly*) as a way of expressing that a formula is true no matter what world is taken as actual. The reader puzzled by the previous formula is confusing it with $\textcircled{1}A\leftrightarrow\boxed{2}\ \textcircled{1}A$, which is not valid (even in the weaker sense). Furthermore, we can understand the right side of this biconditional, or its equivalent $\boxed{\bigcirc}$A (read here as *fixedly actually* A), to represent something close to Evans' *deep necessity*, and $\boxed{1}$A to represent his *superficial necessity*. Evans' examples of these notions employed the device (which may or may not be exemplified in natural language) of *descriptive names*. A descriptive name for x is a name that behaves semantically like the definite description *the object that is actually P*, where P is a predicate that holds only of x and *actually* interpreted as ①. Thus descriptive names are rigid (i.e., they have fixed denotations) with respect to e-worlds, but not u-worlds, whereas ordinary proper names are said to be rigid with respect to both worlds and ordinary definite descriptions (e.g., "the tallest woman in the room,") may be rigid with respect to neither. We may call a description formed with the *actually* operator in this way a *rigidified* description. If **λ** is the descriptive name associated with *the actual length of the standard meter bar* then the sentence *if there is a unique length of the standard meter bar then **λ** is the length of the standard meter bar* is deeply but not superficially necessary. The sentence *if there is a unique length of the standard meter bar then the length of the standard meter bar is the length of the standard meter bar*, however, is both superficially and deeply necessary. Within Davies' and Humberstone's framework, we can understand Evans' distinction as saying that the first sentence is true along the diagonal, i.e., "whichever world had been actual, [the sentence] would have been true at that world considered as actual," while the second is true along the horizontal, i.e., from the perspective of the actual world as actual, the sentence is true at every world. In logical notation, A is deeply necessary with respect to pair (u,v) and model M if $(u,v)\vDash_M \boxed{\bigcirc}A$ and superficially necessary with respect to (u,v) and M if $(u,v)\vDash_M \boxed{1}A$. Since $\boxed{\bigcirc}A$ is true at one pair iff it is true at all, can we take deep necessity to be a property of sentences relative to models. We can similarly take superficial necessity to be a property of sentences relative to models and u-worlds. In applying these notions to natural language, we understand superficial necessity *simpliciter* as superficial necessity with respect to the actual world.

The informed reader will have already noted the similarity between the example here of a deeply, but not superficially, necessary sentence and the example in Kripke 1972 of a sentence expressing a contingent truth knowable *apriori*. Indeed, once we see how the example is constructed, the framework of Davies and Humberstone allows the construction of even simpler "toy" examples of the phenomenon. $\boxed{\bigcirc}(P\leftrightarrow \textcircled{1}P)$ is strongly valid, but $\boxed{1}(P\leftrightarrow \textcircled{1}P)$ is not even weakly valid, so sentences of the form $(P\leftrightarrow \textcircled{1}P)$ are all deeply, but not superficially, necessary sentences. Indeed, Davies and Humberstone tell us, "one can know *apriori* that grass is actually green iff grass is green." Kripke's and Evans' examples can be replaced by ones requiring no special views about the semantics of proper names. Davies and Humberstone are cautious about the degree to which deep necessity and *apriority* coincide. If true proper names are rigid with respect to u-worlds and e-worlds, then true identity and difference statements employing them must be deeply necessary, yet a plausible case can be made that some of these are not knowable without observation. In the other direction, other interpretations of the 2D framework allow arguments that sentences true along the diagonal are knowable *apriori*, but Davies and Humberstone are content to remark that they have yet to find an example representable in their formal language that is not. Subsequent work has shown less restraint.

In a similar way, we can construct toy examples of the necessary empirical. Since *Grass is green* is true at (u_0,u_0) where u_0 represents the actual world, *grass is actually green* is true at (u_0,v) for every possible world v, and so it is superficially, but not deeply, necessary. Since there is no way to know that this sentence is true without knowing that grass is green, we have a simple example of a necessary truth, not knowable *apriori*. Davies and Humberstone consider, without endorsement, suggestions that philosophically more interesting examples might have similar structure. *Water is* H_2O (for chemically naive speakers). *Red objects reflect light of wavelength about 650nm*. *Actions possessing a property that actually arouses in me a feeling of disapproval are wrong*. The subject of the sentence in each case can be regarded as containing an explicit or implicit *actuality* operator. So understood, each may be seen as superficially, but not deeply necessary, and as necessary but not knowable *apriori*.

When a sentence is asserted, we may take it as asserting the truth of the sentence at the actual world from the perspective of that world considered as actual. If the connectives of 2D logic represent logical form, the two notions of validity correspond to two common characterizations of logical truth. A sentence is valid if it is true (i.e., it can be truly asserted) in virtue of its logical form. It is strongly valid if it is necessarily true in virtue of its logical form. Thus, this interpretation of the 2D framework, if correct, would show that the two characterizations come apart. Another useful distinction, somewhat related, is made by Davies and Humberstone (with attribution to Michael Dummett) and emphasized in Lewis 1981. Considered as assertions, our two sentences about the meter bar and, more generally, two sentences of the form P and ①P, apparently say the same thing. Yet each contributes differently the truth of sentences containing them. For $\boxed{1}$(grass is green) is false, while $\boxed{1}$①(grass is green) is true. We may say that A and B have the same *assertive content* (relative to M) if they agree on the diagonal, i.e., if $\vDash_M \boxed{\bigcirc}(A \leftrightarrow B)$. They have the same *ingredient* sense, if they agree at all pairs,

i.e., I ff $\vDash_M \boxed{1}\ \boxed{2}\ (A \leftrightarrow B)$.

Although Davies and Humberstone are relatively cautious in their claims on behalf of two-dimensionalism, it may be useful to list a few questions about their interpretation of the logical machinery. Analogous questions often apply to interpretations discussed in subsequent sections.

1. Is the interpretation of $(u,v)\vDash_M A$ (and the role of the u-world in particular) coherent? How

should we understand locutions like *true at v from the perspective of u considered as actual*, and *if the actual world were u*. It is not obvious that these come to the same thing or that the second describes an entertainable condition. Perhaps the motivating "intuition" that another world might have been actual is confused and ought to be discouraged rather than represented.

2. Is the set of u-worlds and the set of e-worlds really the same? In answering question one, we may determine that not every possible world might have been actual or that the objects we might "consider" as possible worlds are not possible worlds at all. (See discussion of Chalmers, below.) We then lose the homogeneity characteristic of 2D logic that made talk of the diagonal sensible.

3. Are the explanations of the necessary-empirical and the contingent-*apriori* plausible? In the Davies-Humberstone framework, the idea underpinning these explanations is that these sentences contain, explicitly or implicitly, something like an *actuality* operator. Notice also that the puzzling pairs of properties are attributed to sentences,whereas Kripke spoke of *statements*.

KAPLAN

David Kaplan is routinely cited as a father of two-dimensionalism, but the logical machinery employed in Kaplan 1978 and 1979, and its motivation and interpretation, are quite distinct from those described in other sections of this survey (and he seems anxious to distance himself from the enterprise--1989 p 512). The main concern in the Kaplan papers is the logical and semantic properties of *indexical* expressions like *I, here*, *now* and the lessons that these might have for nature of meaning generally. In particular, Kaplan observes that the outlook outlined in the first paragraph here, whereby traditional modal logic is generalized by simply replacing possible worlds with n-tuples of parameters, will fail to capture the sense in which a sentence like *I am here now*, is deeply and universally true. If we restrict the n-tuples to the “proper” ones, say the 4-tuples (w, a, p, t) where agent a in world w is producing a linguistic expression at place p and time t, we will falsely represent the sentence as necessarily true. If, on the other hand, we allow the parameters to vary independently, we will improperly suggest that it can be uttered falsely. The solution is to recognize that the parameters here can play two roles. They can be features of *context* in which an expression is produced or the *circumstances* in which the content expressed by an expression in such a context is evaluated. The content of an utterance of *I am here* by David Kaplan in Portland (in the actual world) on March 26th 1977, for example, is the proposition that Kaplan was in Portland then. Since this proposition is false when evaluated in circumstances where he was elsewhere on that date, it is not necessary. Contents are supposed to be “what is said” by an expression in a context. They can be conveniently represented by functions from circumstances to extensions, though we should remember that such functions are not identical to contents they represent. We can similarly represent another variety of meaning, which Kaplan calls *character* by functions from contexts to contents.

If we take possible worlds as the sole features of context and circumstance, the logic of demonstratives in Kaplan 1978 is built on the 2D logic described here with connectives ① (*actually*) and $\boxed{1}$ (*necessarily*) and validity understood in the weaker sense. Because Kaplan’s version is formulated within predicate, rather than propositional, logic, however, he is able to introduce a formal device by which the rigidified descriptions of Davies and Humberstone (and therefore the descriptive names of Evans) can be represented. For any term **t**, **dthat t**, is a term whose extension at (u,v) is the extension of **t** at (u,u). Thus, if **s** represents *the length of the standard meter bar*, then **dthat s** represents the actual length of the standard meter bar. What Kaplan's logic obviously *omits* is anything like the connectives $\boxed{2}$ (*fixedly*) and $\boxed{\bigcirc}$ (*fixedly actually*), that feature so prominently in the discussion of Davis and Humberstone. If the interpretation of $\bigcirc$**A** at a pair (u,v) depends on the interpretation of A at pairs (u′,w) for u≠u′, then $\bigcirc$ is operating on the character of A rather than merely its content. Kaplan 1989 labels such operators, and the expressions that they might represent, *monsters*, and maintains that English (unless being used metalinguistically) has no such devices. Although Davies and Humberstone's *fixedly* is not an expression of ordinary English and Kaplan's *I*, *here* and *now* are not monsters, the basis for the general claim is somewhat unclear and it has been questioned by a number of authors. (See, for example, Schlenker 2003.)

Kaplan does not emphasize the connections between this framework and the contingent *apriori* or the necessary empirical. Kaplan 1978 (p 85) alludes to a "structural" similarity between his character/content distinction and Kripke's *apriori*/necessary one. Kaplan 1989 (p550) mentions examples reminiscent of those in Davies and Humberstone. If **s** and **t** are definite**,** non-rigid descriptions for which **s**=**t** is a contingent, empirical truth, then **dthat s** = **dthat t** is both necessary (because all identity statements with rigid designators are necessary) and empirical (because it is true iff **s**=**t**). On the other hand, **s** = **dthat s** is both contingent (because **s** is non-rigid) and knowable *apriori* (because it is valid in the logic of demonstratives). The argument for this last claim would seem to indicate that the contingent *apriori* extends beyond the cases mentioned by Kripke to include examples like Kaplan's *I am here now*. Kaplan nowhere makes such an observation, perhaps because the considerations raised in question three of the previous section are so salient for these examples. *I am here now* is valid in Kaplan's logic, and so we can know *apriori* that it is true. Its content in a given u-world, however, is something empirical, like the proposition that Kaplan was in Portland in 1977. While we might know *apriori* that the sentence *I am here now* is true, it requires empirical knowledge to know what proposition the sentence expresses in a particular context. Soames 2005 maintains that all of Kaplan's examples of puzzling pairs (but not Kripke's) are defective because the proposition alleged to be *apriori* is not the same as that alleged to be empirical (and that the citations above represent a misleading and unfortunate digression in Kaplan's otherwise insightful observations). Lewis 1994, and other of what Soames calls "strong" two-dimensionalists, however, maintain that all alleged examples of puzzling pairs have this feature.

LEWIS

In an earlier paper (1981), Lewis suggested that the idea that there is some intuitive, univocal notion of "what is said" captured by Kaplan's content is illusory. On the Lewis 1981 version of two-dimensionalism, the appropriate distinction is not between features of context and features of circumstance, but rather between context as a whole and a small package (or *index*) of contextual features that are independently shiftable. Every sentence gets a truth value at a context, but, for a compound sentence, that truth value depends on the truth values of its subsentences at indices in which certain contextual features have been shifted (and to the mode of compounding in question). *I have been here* is a compound of the sentence *I am here* using the *it has been that* mode. The truth value of an utterance of *I have been here* in a context with Kaplan as speaker in Portland in 1977 is determined by the truth values of *I am here* at indices in which the time coordinate of that context is shifted toward the past. After the coordinate has shifted, the index may correspond to no context at all. (Kaplan may not be there then.) Since any sentence may occur either alone or as part of a compound, we must evaluate all sentences relative to both contexts and indices. These are the two dimensions. From this perspective, the absence of monsters is a matter of stipulation. If there is any operator ◯ for which the truth of ◯A in a context c depends on the truth of A when some feature of c has shifted then that feature is, by definition, part of the index. This does not rule out the possibility that two different operators might make use of the same contextual feature. In that case indices may require two coordinates to store different values of that feature. Perhaps this situation would be as monstrous as the presence of the context-altering creatures that frightened Kaplan.

STALNAKER

For Stalnaker, the need for 2D logic arises in connection with an explanation of *assertion*. We start with the familiar idea that the content of an assertion, a proposition, can be roughly identified with a set of possible worlds (the worlds in which the proposition is true). Each participant brings to a conversation a set of *presuppositions*, i.e., a set of propositions **p** such that he is disposed to act as if he takes **p** to be true, takes the others to take it to be true as well, takes them to take him to take it to be true, and so on. The intersection of these propositions, the speaker's *context set*, will comprise all worlds that the speaker takes to be "live options" in the conversation. The participants in a conversation value communication. Since communication is impeded when presuppositions diverge, context sets will tend to become *non-defective*, i.e., identical for all. Given a non-defective context, conversants have a further common interest in narrowing it, that is, in more fully specifying the way the world is taken to be, though if they disagree about the facts their interests in how this should be done may conflict. An assertion of the proposition **p** is, in essence, a proposal to narrow the context set by eliminating all worlds where **p** is false (or, equivalently, by selecting just those making **p** true.)

One difficulty with this very plausible picture is that necessary propositions are nowhere false, and so cannot narrow any context set. Yet utterances like *Hesperus is identical to Phosphorus, it is now three o'clock*, and an *ophthamologist is an eye doctor* appear to be respectable assertions of just such propositions. These appearances, Stalnaker contends, are deceiving. *Hesperus is Phosphorus* does *standardly* express a necessary proposition. Since asserting a necessary proposition fails to narrow the context set, however, it would violate a fundamental principle of conversation. We are therefore justified in interpreting one who appears to be doing so as asserting another instead. (Stalnaker is here invoking ideas of Paul Grice. See 4.3** of this volume.) That *Hesperus is Phosphorus* expresses a necessary proposition follows from the fact that *Hesperus* and *Phosphorus* rigidly denote the planets visible in the evening and morning sky and that these are one and the same. The speaker who appears to assert that Hesperus is Phosphorus, it is reasonable to suppose, fails to presuppose this last condition. His context set includes a world w in which the morning and evening appearances are of a single planet and a world ′in which they are of different planets. In w his utterance expresses a necessary truth; in w′, a necessary falsehood. The situation can be modeled in the now familiar way by a 2D model M, where $(u,v)\vDash_M A$ indicates that what A expresses in u is true in v. If the truth values of A are arranged geometrically as described above, the row of truth values at vertical position u represents the proposition expressed by A in u. The speaker is proposing to eliminate w′, but not w, from the context set. He cannot do this by asserting propositions that would be expressed by A or ¬A in w or w′, for each of those would eliminate either nothing or everything. He *can* do it by asserting the proposition expressed in either world by ①A (†A in Stalnaker's notation). ①A can be regarded as expressing, at every world that the proposition expressed by *Hesperus is Phosphorus* is true, or that the denotations of *Hesperus* and *Phosphorus* are identical. In general, it is often reasonable to interpret what appears to be an assertion of a necessarily true proposition that A as instead an assertion of a contingent proposition that ①A.

Stalnaker 2004 emphasizes that he is not suggesting a semantical theory under which the proper name *Hesperus*, like Kaplan's indexicals *I* and *here*, gets different interpretations in different contexts. Semantical considerations still determine that *Hesperus is Phosphorus* standardly expresses either a necessarily true proposition or a necessarily false one, and empirical considerations help us determine that it is the former. Pragmatic considerations that presuppose and utilize the standard interpretation, however, may lead us to conclude that an utterance of that sentence is an assertion of an entirely different, contingent proposition. Utterances of *Hesperus* do denote different individuals in different u-worlds, but this should not be thought of as reflecting the meaning of *Hesperus*. Under appropriate conditions the context set might contain a world where such an utterance denotes a horse or a real number.

Similar considerations suggest that Stalnaker's two-dimensionalism, though perhaps explaining how sentences that standardly expresses necessary propositions can convey information about the world, has little to tell us about the existence and nature of the contingent *apriori* or the necessary empirical. Stalnaker 1981 remarks that $\boxed{\circ}$ can be understood as an *apriori* truth operator, but Stalnaker 1989 rightly expresses strong misgivings. If it is sometimes appropriate to interpret *Hesperus is Phosphorus* as expressing something necessarily false, it may be similarly appropriate to do so for *7+5=12*, *Tuesday follows Monday* or any other standardly *apriori* sentence. If so then these will be false somewhere on the diagonal and an application of $\boxed{\circ}$ will produce sentences everywhere false.

CHALMERS

The currency of the term *two-dimensionalism* is largely due to David Chalmers, the subject's most voluble and energetic expositor. Yet, on Chalmer's preferred interpretation, 2D logic, as described here, is largely hidden from view. Having climbed the 2D ladder, Chalmers is happy to put it aside. (Some qualification is warranted regarding this and other remarks in this section. A large proportion of Chalmer's writings on two-dimensionalsm is devoted to interpretations other than what I call his preferred interpretation. "Semantic pluralism" allows him to regard some of these as viable alternatives, capturing notions of meaning distinct from his own primary concern. Others are presented as reformulations of the preferred framework, taking different notions as primitive than the preferred framework but, under appropriate philosophical assumptions, arguably equivalent to it. The intention is to avoid losing the reader who might have scruples about his choice of primitives. My remarks concern the preferred interpretation. A further warning: in this section more than others, qualifications and refinements are omitted to convey basic ideas in reasonable space.)

Chalmers sees two-dimensionalism as clarifying connections between cognitive content and necessity revealed to us by Frege and Carnap's discussions of *sense* and *intension*, but subsequently obscured by Kripke's persuasive examples of necessary empirical statements. For this reason, he deliberately seeks an interpretation satisfying a condition like the one Stalnaker repudiated. He takes u-worlds to be *scenarios*, rather than possible worlds, corresponding to "ways that the world might turn out to be, for all we know *apriori*." On the preferred analysis, scenarios are identified with equivalence classes of complete sentences in some idealized (infinitary) language capable of representing every hypothesis about the world. A complete sentence is one that is epistemically possible (this notion is primitive) and implies S or ¬S for every sentence S in the language. Sentences within equivalence classes all imply each other. Some scenarios imply ("verify") utterances of *Hesperus is Phosphorus*, others verify utterances of *Hesperus is not Phosphorus*, but none verifies the conjunction of these and none verifies *Venus is not Venus*. A scenario verifying *Hesperus is not Phosphorus* might have that sentence as a conjunct, but it might instead have *the bright celestial objects visible in the morning and evening are Mars and Mercury, respectively*, or perhaps *no celestial object appears in both the morning* *and evening*. In general, Chalmers argues, for any scenario u, there will be a “relatively limited” vocabulary V such that if u verifies an utterance S, then u verifies some sentence D containing only expressions of V that implies S. There is no requirement that expressions of V be qualitative and no requirement that objects mentioned in s be *definable* in V.

Since scenarios and worlds are distinct, this conception affords no notion of a *diagonal*. Instead, Chalmers defines the “1-intension” of a sentence S to be the function that assigns to each scenario u the value **T** if u implies S and **F** if it implies ¬S. It now becomes plausible to argue that S is *apriori* if and only if its 1-intension always takes the value **T**. Though it is unclear that his preferred interpretation requires it, Chalmers argues that there is a *correspondence* between scenarios and worlds (or *centered* worlds, i.e., worlds with time, place and speaker marked) so that the 1-intension corresponds to a diagonal after all, but he insists that it should not be seen as deriving from it. The “2-intension” is just the familiar function from possible worlds to truth values. Thus, sentences are not (or not fundamentally) evaluated at scenario-world pairs, but at worlds and scenarios alone. The intuition is that an utterance of S is *verified* by *scenario* u if it is true if the actual world turns out to really *be* as described by u. It is *satisfied* by *possible world* v if it would be true if the actual world *were* as described by v. The scenario according to which no body is visible morning and evening verifies *Hesperus is not Phosphorus,* but the world with no such body still satisfies *Hesperus is Phosphorus*. If the interpretation has the properties it was designed to, then all the puzzling pair examples have similar structures. An utterance is true *apriori* if and only if its 1-intension takes the value **T** at all scenarios and it is necessary if and only if its 2-intension takes the value **T** at all worlds. It should not be surprising that each of these conditions can obtain without the other. Whether that implies that there are necessary empirical and contingent *apriori* propositions depends on whether one takes the 1-intension and 2-intension of an utterance to correspond to two propositions, both expressed by that utterance, or whether one takes them to be parts of a single proposition expressed by it. Chalmers leans toward the latter. To say that puzzling-pair examples have epistemological and metaphysical properties in virtue of different aspects of their meanings does seem more reasonable than to insist they express two propositions.

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FURTHER READING

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Marx, M. and Y. Venema 1997, Multi-Dimensional Modal Logic, Kluwer, Dordrecht Academic Publishers. (The only book-length treatment of many-dimensional logics. Contains a chapter on two-dimensional logic with a wealth of technical results on logical systems with connectives like those described here—but not the normal form and translation results sketched above.)

Soames, S. (2005), Reference and Description: The Case Against Two-Dimensionalism, Princeton: Princeton University Press. (A detailed and sustained attack on two-dimensionalism, which the author sees as a misguided attempt to revive Fregean descriptivism in the face of compelling arguments by Saul Kripke against it. If Chalmers is the subject's most energetic expositor, Soames is its most energetic critic.)

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Stalnaker 2004, it contains a useful introduction by the editors, a shortened version of Chalmers 2006, papers by David Braddon-Mitchell, Martin Davies, Frank Jackson, Frederick Kroon, Michaelis Michael, Philip Pettit, Laura Schroeter, and a posthumous publication of Gareth Evans' comments to Davies and Humberstone concerning their 1980 paper.