

# Necessary, Transcendental and Universal Truth

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## Abstract

How is it that we judge it absurd that Socrates might be human and not exist, yet true that he is necessarily human and true that he might not exist? The answer, Fine tells us, has profound implications for our understanding of the concepts of existence, identity and modality. It requires that we distinguish between *worldly* sentences, whose truth values depend on circumstances and *unworldly* ones, which are true or false independently of circumstances. Unworldly sentences, like *Socrates is human*, express *transcendental propositions*. Although these are not, strictly speaking, true in every (or indeed in any) possible world, we accept them as necessary in an extended sense. Unless the context gives us special reason, however, we are reluctant to extend the concepts of necessity and possibility further to include worldly-unworldly “hybrids” like *Socrates is human and does not exist*. I argue that this understanding of the relation between necessary and transcendental truth is backwards, and perhaps contrary to what Fine himself has elsewhere advocated. What is taken for (unextended) necessity in the puzzle analysis, I call *universal truth* and I suggest that universal and transcendental truths are both necessary. To further clarify this view I present a simple formal system with distinct operators for necessary, transcendental and universal truth. It turns out that the logic for universal truth coincides with something that Arthur Prior had once labeled *System A*. With the benefit of now-familiar techniques, it is shown that Prior’s conjectured axiomatization for this system is correct. Finally, the formal system is enriched by the addition of an operator for *actually true*. This raises philosophical questions that sharpen understanding of the worldly/unworldly distinction. The extended system is axiomatized and shown to correspond to the system **S5A** of Crossley and Humberstone in much the same way that the system without actuality corresponds to **S5**. The new logical systems lose the simplicity of connectives that apply uniformly to all sentences, but gain simpler axioms and greater fidelity to the notions formalized.

## 1. Introduction

Thirty-nine years ago, I was a graduate student at Stanford approaching the critical time for choosing a dissertation topic. I knew I wanted to write something having to do with the modal logic I had been learning in graduate school, ideally something that would call for both

technical work in logic and its application to interesting philosophical problems. I was excited that a talented and friendly young Englishman who seemed to share my interests was visiting the Department. I don't remember exactly when I had the thesis-topic conversation with Kit, but I do remember his offering to take a look at the papers I had been writing. I showed him some of my efforts in tense logic and he pointed out that there was a kind of theme present--in every case I argued that fidelity to the phenomena for which I sought formal treatment required that formulas be sorted, so that not all connectives should apply to all formulas uniformly. Sometimes this was because it was natural to restrict the interpretations of different sorts of formulas in different ways: *John swims IN the Channel* should be true at a temporal interval only if it is true at all its subintervals; *John swims ACROSS the Channel* should be false at the subintervals of those where it is true. Sometimes it was because the sentences represented were naturally to be regarded as true or false with respect to different sets of parameters. Some sentences are naturally thought of as being evaluated at instants of time, others at intervals of time, and still others are naturally thought of as assuming truth values independently of time altogether. So, at Kit's suggestion, I tried to gather some of these examples together with a few others as part of a general study of many-sorted modal logics. As what I imagined to be a very simple example, I briefly considered an application to the standard alethic modal logics. There were two basic sorts of formulas: one world-relative and one world-independent, and a third sort comprised of Boolean combinations of these two. The necessity operator applied only to formulas of the first and third sort to produce a formula of the second.

In retrospect, I am somewhat embarrassed by the lack of attention I paid to the metaphysical underpinnings of these distinctions. I cringe particularly at a sentence promising "The next two chapters [after the one containing these remarks about necessity] deal with more serious applications." Many years after my dissertation was written, I watched Kit turn his attention to very similar ideas in a lecture at the University of Maryland entitled "Necessity and Non-Existence."<sup>1</sup> Knowing his philosophical temperament as I do now, it is not surprising to me that he *did* take the metaphysical matters seriously and had a great deal to say about them. I am grateful for the opportunity now to reflect more deeply about both the metaphysical and logical aspects of these matters, and on what Kit had to say about them.<sup>2</sup>

"Necessity and Non-Existence" defends the view that there is a distinction between worldly and unworldly sentences that is similar to a more familiar<sup>3</sup> distinction between tensed and tenseless sentences. *Socrates exists* is true at some times and false at the others. As a consequence, *Either Socrates exists or Socrates does not exist* is true at all times. It is, in Fine's

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<sup>1</sup>This paper has since been published under the same title in [Fine, 2005], pp321-356.

<sup>2</sup>I am further pleased by the circumstance that this reflection, as the reader will see, has also led me back to the ideas of Kit's teacher (and my grand-teacher?) Arthur Prior.

<sup>3</sup>To say this distinction is familiar is not to say that it is universally accepted among philosophers. Fine argues, however, that even those whose acceptance of McTaggart's A-theory of time leads them to reject the thesis that *Socrates exists* is true at a time in a way that *Socrates is self-identical* is not, should be willing to grant that only the former is true *because* of how things are at the time. Thus they should grant the distinction even if they understand it differently.

terminology, *sempiternal*. By contrast, *Socrates is self-identical* and *Socrates is human*<sup>4</sup> are true *regardless* of time. Fine calls these *eternal* truths.<sup>5</sup> To conform more closely to contemporary usage it might better to use *timeless* here, and *timely* for those sentences, including the sempiternal ones, whose truth depends on time. Similarly, *Socrates exists* and *Socrates exists or Socrates does not exist* are worldly truths. They are true or false because of the circumstances of the world, because of how things have turned out. The second is true in every circumstance, i.e., it is true no matter how things turn out. It is necessarily true. By contrast, *Socrates is self-identical* and *Socrates is human* are completely independent of circumstance. They are unworldly sentences and the truths they express are *transcendental*.

Once the distinction between worldly and unworldly is acknowledged among sentences, it is natural to extend it to other domains. Predicates like *is human* and *is self-identical*, which combine with names to form unworldly sentences, are themselves unworldly; those like *is snub-nosed* and *sits*, for which similar combinations form worldly sentences, are worldly.<sup>6</sup> Properties

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<sup>4</sup>Fine's example is *Socrates is a man*, which is supposed to correctly predicate a substance sortal property of an individual. I substitute *human* for *a man* because the latter may inappropriately suggest *male* or *adult* (in which case it would not name a substance sortal). Fine prefers his wording because of qualms about whether *being human* might indicate belonging to a biological species and whether such belonging constitutes having a substance sortal property. (See [Wetzel] for an argument that it does not.) Fine (private communication) has suggested *Socrates is a human being* might be a better alternative than *Socrates is human*. I have resisted the suggestion because of worries that the former sentence is more likely to be read as having existential import. All these issues are peripheral to those addressed in the paper and the reader should feel free to substitute any example that does correctly predicate a substance sortal of an individual. The reader who is generally skeptical about substance sortals may stick to examples like *Socrates is self-identical*.

<sup>5</sup> In employing this terminology, Fine evokes old metaphysical distinctions. See, for example, [Kneale]. But in the older discussions, the labels apply at the most fundamental level to beings or things rather than truths or sentences.

<sup>6</sup>Linda Wetzel has made the interesting suggestion that *sitting* and *snub-nosed* might be unworldly predicates when applied to Rodin's *The Thinker* and the Vatican's bust of Socrates. These examples would need to be examined with some care. It is not really the sculptures that are sitting or snub-nosed but their subjects. The subject of the Vatican bust would seem to be Socrates, for whom being snubnosed was a worldly property. As for Rodin's subject, he appears to be sitting temporarily to facilitate contemplation, rather than suffering some incurable metaphysical paralysis. Regardless of whether these examples can be understood in a way that makes them true, they raise questions about whether predicates have stable "signatures" and, more generally, whether properties and relations should be seen as sources for the unworldliness of propositions. It is reasonable to suppose that individuals may have both worldly and unworldly properties. The worldly Socrates, for example, has the worldly property of snubnosedness and unworldly property of self-identity. Similarly, the unworldly number 2 is both self-identical and the number of Adam and Eve's children. Is there any reason why some properties might not, as Wetzel suggests, be similarly fickle? An example of a sort different than Wetzel's might be the following: *four is smaller than five* is unworldly, whereas *David is smaller than Goliath* is

like *humanness* and *self-identity*, expressed by unworldly predicates, are unworldly; those like *being snub-nosed* and *sitting* are worldly. Individuals like Socrates have worldly existence; numbers and sets have unworldly existence. Facts that can be expressed by worldly sentences are worldly; those that can be expressed only by unworldly sentences are unworldly. Unworldly facts, individuals and properties belong to a *transcendental* realm of reality.

It is one thing to acknowledge the distinction between worldly and unworldly and another to agree on a particular inventory of the transcendental realm: the “worldly” philosopher may grant that the distinction makes sense, while insisting that all facts are determined by circumstances, and therefore that the transcendental realm of reality is empty. While Fine obviously thinks the worldly philosopher is wrong, “Necessity and Non-existence” contains only the very beginnings of speculation about what kinds of facts might populate the transcendental realm. It contains just enough to address a simple-seeming puzzle whose answer, Fine tells us, “has profound implications for our understanding of the concepts of existence, identity and modality and for how these concepts connect to one another and to the world.”<sup>7</sup>

I believe that Fine mischaracterizes the relation between necessity and transcendental truth and that this undermines his solution to his puzzle. In Section 2 below I summarize Fine’s puzzle and his preferred solution. I explain the weakness I see in the solution and the mistaken view of the necessary/transcendental relation that underlies it and I offer an alternative view of that relation. Section 3 outlines a simple formal system with distinct operators for necessary truth, transcendental truth and truth in all worlds to help clarify the alternative view. This system is tantamount to one discussed many years ago by Arthur Prior as “The System A.”<sup>8</sup> In Section 4, I pause to consider my system and Prior’s system A in more detail. In Section 5, I consider extending my system by the addition of an “actuality” operator. That extension brings to light some additional questions about how the worldly/unworldly distinction should be made and raises other questions of philosophical interest. Section 6 is a brief summary and conclusion.

## **2. Could Socrates be both human and non-existent?**

The puzzle serves both as an illustration and an application of Fine’s ideas on worldly and unworldly truth. Consider the following simple argument.

- (P1) It is necessary that Socrates is human;
- (P2) It is possible that Socrates does not exist;
- (C) Therefore it is possible that Socrates is human and does not exist.

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worldly. Here a plausible reply is that the sentences equivocate between distinct worldly and unworldly relations. The classification of predicates mentioned above suggests that Fine believes such responses will always be forthcoming—that no such examples are possible. Nothing in his writing, however, would seem to commit him to the view that there cannot be some predicates that lack stable signatures.

<sup>7</sup> [Fine, 2005] p 321.

<sup>8</sup>See [Prior], pp 133-139 (“Appendix C”).

Taken alone, Fine thinks that the conclusion should strike us as absurd. The second premise, however, is obviously true—since it is possible that all humans, both individually and as a group, fail to exist it is surely possible that Socrates fails to exist. The first premise seems equally true—being human is a part of Socrates’ nature, and so it must be necessary that he be human. The conclusion follows, as Fine says, by “impeccable modal reasoning,” reasoning that is codified in the weakest of plausible alethic modal systems. So we have an apparently absurd conclusion following by apparently valid reasoning from apparently true premises—i.e., a paradox.

We might think that the paradox can be resolved by invoking distinctions commonly made in discussions of necessity and non-existence. For example, Arthur Prior’s system Q adopts a view according to which sentences mentioning merely possible individuals lack truth value, and necessity may be strong (truth in all worlds) or weak (falsity in no world). More standard semantics for modal systems retain bivalence and distinguish between unqualified necessity (truth in all worlds) and qualified necessity (truth in all worlds containing the individuals mentioned). It is natural to think that Fine’s puzzle argument involves an equivocation between strong and weak modalities or one between qualified and unqualified modalities, but he makes persuasive arguments that such solutions are unsatisfactory.<sup>9</sup>

Fine’s own solution invokes the worldly/unworldly distinction. If we take necessity to be truth at all worlds or circumstances, then the first premise, that Socrates is necessarily human, is either meaningless or false. Socrates is not human in every circumstance; he is human independently of circumstances. It is quite natural, however, to *extend* the notion of necessity to include sentences of both types. (At the same time, we should extend the notion of possibility to include sentences that are unworldly and true as well as those true at some world.) In this extended sense, we can accept that Socrates is necessarily human. We have some reluctance to invoke the extended sense of necessity, however, so there is also some tendency to take the predicate *human* to mean the same as some worldly approximation—*existing human* being the most likely candidate. This understanding allows us to accept the first premise even on the unextended sense of necessity. The second premise, that it is possible Socrates does not exist, is true in either the unextended or extended sense of possibility, so we can happily accept that as well. The conclusion, however, applies possibility to a *hybrid* sentence: *Socrates is human and does not exist* is a boolean combination of worldly and unworldly parts. It is possible to further extend the notion of necessity so that it applies not only to worldly and unworldly sentences, but to these hybrids as well. Fine calls this the *superextended* sense of necessity, and he thinks we are naturally even less inclined to adopt it than the extended sense. The reasons for our reluctance, Fine says, are hard to articulate, but we can get a sense of the phenomenon by considering an analogy in the temporal realm. We are naturally “uncomfortable” with the idea that it will be the case that dawn breaks and two plus two equals four. We may have

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<sup>9</sup>For a defense of an “orthodox” solution based on the distinction between weak and strong modalities that attempts to overcome Fine’s objections, see [Forbes], pp 285-286. Notice, however, that denying P1 on the grounds that Socrates (and indeed all of humanity) might not have existed seems to be presupposing that *Socrates is human* is false *in those circumstances*, which is exactly what Fine denies.

accommodated ourselves to the superextended interpretations of modal and tense operators, Fine suggests, because of “scientific” benefits of doing so. In applying these operators to hybrid sentences we can say things that might be difficult or impossible to do otherwise. Given these various senses of modalities, the puzzle argument can be interpreted in a variety of ways.

Here are three. 1. We can misconstrue *human* as worldly and take necessity and possibility in the unextended sense throughout the argument. In that case, premise one is well formed and the argument is valid. Both premise and conclusion, however, are false. 2. We can understand *human* correctly, and take necessity and possibility in the extended sense throughout. In this case the premises are true but the conclusion is ill formed. 3. We can understand the predicates correctly, but take necessity and possibility in their superextended senses. In this case the argument is sound and valid. Its air of paradox is explained by the observation that, in the absence of motivation provided by the argument, we are unlikely to interpret the necessity operator in the conclusion in the super-extended sense.

There are, I think, two weaknesses in this account that are closely related. The first concerns the idea that, in applying necessity to hybrids of worldly and unworldly sentences we invoke a “superextended” notion of necessity that is further removed from the ordinary notion of necessity as truth in all possible worlds and considerably more difficult for us to accept than the application of that notion to unworldly sentences themselves. Fine himself suggests that in making the transition to the extended sense we are already admitting a new, “degenerate” way that a sentence may be true at a world; it may be true at a world “simply because it is true regardless of how things are in the world.” Once we admit that both worldly and unworldly sentences can be regarded as true or false at a world, however, there would seem to be no obstacle to so regarding their truth-functional combinations. We know very well how to compute the truth-value of a conjunction at a world when we have the truth-values of its conjuncts at that world. Knowing that truth values of conjunctions in possible worlds are computable, it should make perfect sense to ask whether these conjunctions are true in all or some possible world.

Such combinations, moreover, would seem to clearly fit our criterion of worldliness. Their truth depends on the worldly circumstances. It is hard to imagine why one who understood that the sentence *Socrates is an existent human* depends for its truth on worldly circumstances would balk at the notion that *Socrates exists and is human* should depend similarly on such circumstances. If such truth-functional combinations are worldly, then understanding how necessity and possibility applies to them should be unproblematic. Similar remarks apply in the temporal realm. It is true that *Dawn breaks and  $2+2=4$*  is an odd sentence. But many homogeneous conjunctions are equally odd: *Socrates is self-identical and nine is less than 10*. *The universe expands and Cain killed Abel*. Furthermore more natural heterogeneous compounds are not difficult to find: *If that creature is traveling at thirty miles per hour, it is not human*. *Either Wiles made a mistake in his proof or Fermat’s Last Theorem is true*. These seem perfectly reasonable things to say, requiring no special use of *if* and *or*.

The more fundamental weakness is Fine’s uncritical acceptance of the common idea that necessity is truth at all possible worlds. This acceptance is somewhat surprising. In the introduction to the volume in which “Necessity and Non-existence” was published, Fine writes

that his thinking about modality has been sustained by a “deep animosity” to views of Quine and Lewis, according to which necessity, if it is an intelligible notion at all, must be seen as a kind of regularity. A necessary truth is one that is always or everywhere true; a possible truth, one that is sometimes or somewhere true. The apparent contrast between the spare modal landscape envisaged by Quine and the more florid one Lewis discerns simply reflects a disagreement about the range of the regularities permitted by their ontologies. In view of Fine’s animosity to these views, one wonders why he would take transcendental truths to be necessary only in an “extended” sense. If being human is part of Socrates’ nature, then surely he *must* be human in any ordinary pre-theoretic sense of *must*. One would think that transcendental truths like this are, if anything, *more* necessary than those worldly necessities whose truth derives from one set of circumstances in this world and a different set elsewhere. Indeed, the very fact that the question of their being more necessary is sensible suggests that the ordinary notion of necessity is *not* truth in all worlds.

That thought gains credence from one of Fine’s own glosses on the notion of transcendence.

“We might think of the possible circumstances as being what is subject to variation as we go from possible world to another; and we might think of the transcendental facts as constituting the invariable framework within which the variation takes place. Alternatively, we might think of the possible circumstances as being under God’s control; it is what he decides upon in deciding whether to create one possible world rather than another. Thus he can decide whether Socrates exists or not and so he can do something that will guarantee that Socrates exists or does not exist. But there is nothing he can do that will guarantee that Socrates is self-identical or that  $2+2$  is equal to 4; these are the facts that provide the framework in which he makes the decisions that he does, not the facts yet to be decided.”

One might imagine that by changing the right circumstances, God could make something false that would otherwise be true in all possible worlds, or turn something that had been false only in this possible world into something true in all possible worlds. Even God, however, can’t mess with the transcendental.

Let us call a worldly sentence that is true at every possible world, *universally* true. The view Fine favors is that the more basic and natural form of necessity is universal truth and that transcendental truths are necessary only in an extended sense. The view I want to defend against this is that universal and transcendental truths are two subclasses of our ordinary notion of necessary truth, and perhaps that universality is sometimes weaker or less binding than transcendence. There is evidence that Fine might be sympathetic to a view like this or even a little ambivalent between such a view and the one I attribute to him. The critical remarks about “regularity” accounts of necessity and about what lies within and without of God’s control that have already been mentioned provide some of this evidence. Further support comes from the fact that the kinds of truths labeled “transcendental” in “Necessity and Non-existence” are said in other writings to be *necessary* with no indication that any extension of the fundamental notion is involved. Here is Fine on the relation between essence and necessity.

“Indeed, it seems to me that far from viewing essence as a special case of metaphysical necessity, we should view metaphysical necessity as a special case of essence. For each class of objects, be they concepts or individuals or entities of some other kind, will give rise to its own domain of necessary truths, the truths which flow from the nature of the objects in question. The metaphysically necessary truths can then be identified with the propositions which are true in virtue of the nature of all objects whatever.

Other familiar concepts of necessity (though not all of them) can be understood in a similar manner. The conceptual necessities can be taken to be the propositions which are true in virtue of the nature of all concepts; the logical necessities can be taken to be the propositions which are true in virtue of the nature of all logical concepts; and, more generally, the necessities of a given discipline, such as mathematics or physics, can be taken to be those propositions which are true in virtue of the characteristic concepts and objects of the discipline.”<sup>10</sup>

The truth of the unworldly sentence *Socrates is human* presumably flows from the nature of Socrates and the truth of *Socrates is not identical to Plato* flows from the natures of Socrates and Plato, so these both express metaphysically necessary truths as well as transcendental ones. Likewise, the unworldly  $2+2=4$  is true in virtue of the concepts and objects of mathematics, and so expresses a mathematical necessity as well as a transcendental one. This line of thought is maintained and refined in *Varieties of Necessity*.

Even in “Necessity and Non-existence” there is an admission that “...we are accustomed to operating with an inclusive conception of what is necessary and of what is true in a possible world and so we think of any possible world as being like the actual world in settling the truth-value of every single proposition.”<sup>11</sup> It is also maintained, however, that “we naturally operate with a more restrictive conception of what is necessary and what is true in a possible world.”<sup>12</sup> So, for the Fine of “Necessity and non-existence” it seems that we operate with two conceptions of necessity. The more restrictive conception is what I am now calling universal truth, and the inclusive conception is what I believe to be the ordinary conception of necessity.

### 3. A formal system

We can get a clearer picture of this understanding of the universal, transcendental and necessary, by considering a simple formal language with operators representing these notions and providing truth conditions for each operator. The logic of “it is universally true that,” under this semantics is just the logic that Arthur Prior sought as “The System A.”

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<sup>10</sup> [Fine, (1994)], p9.

<sup>11</sup> Fine [2005] p324.

<sup>12</sup> *Ibid* p325.

Our language, like Prior's, will contain formulas of two sorts. We may think of them as the worldly and the unworldly. In keeping roughly with Prior's conventions, capital letters from the beginning of the alphabet will be used for the former; capital letters from the middle of the alphabet for the latter. With numerical subscripts, these will be regarded as particular sentence letters; without the subscripts, as meta-variables ranging over the appropriate formulas. Letters from the end of the alphabet are meta-variables ranging over formulas of either sort.  $[N]$ ,  $[T]$ , and  $[U]$  are unary connectives corresponding to necessary, transcendental and universal truth. For reasons already outlined, I want to resist the idea that there is any special difficulty in forming truth functional hybrids of worldly and unworldly formulas, or that application of  $[U]$  to such hybrids, requires us to first re-interpret their unworldly constituents as, in some derivative or extended sense, true or false in a world. Accordingly, the binary truth-functional connectives are taken to form unworldly formulas when applied to pairs of unworldly formulas, and to form worldly formulas when one or more of the formulas to which they are applied are worldly.

Most of Fine's discussion of unworldly truth relies on two kinds of examples—mathematical truths, and sentences like *Socrates is human* predicating a substance sortal of an individual. There is, however, another class of putatively unworldly sentences that originally motivated Prior's system A and my own consideration of the many sorted-frameworks, and that impresses Fine sufficiently to remark, "for this reason alone, the worldly view should be given up." These are sentences that express modal facts. Consider sentences asserting that a proposition is true in all possible worlds. In Fine's jargon they assert necessity. For me they assert universality. Their truth does not turn on the circumstances in *this* possible world, but rather on the non-existence of certain *other* worlds. On one understanding of modal discourse, worldly sentences have an implicit unfilled argument place for possible world. The (unextended) modalities quantify over this argument place. Once such quantificational devices are applied, the resulting sentences are no longer worldly. In any event, it seems clear that the universal truth connective should apply only to worldly formulas and produce only unworldly ones. The necessity operator, on the other hand, applies to both worldly and unworldly formulas to form unworldly ones. In the case of  $[T]$  there is some choice. Do we want to say that  $[T]$ (Socrates exists) is false or that it is ill-formed? For the universal operator we were trying to remain faithful to the intuition that to say of an unworldly truth that it is true in all worlds is not to say something false, but to say nothing at all. There does not seem to be the same pull in the case of transcendental truth. To say of a worldly truth that it is transcendentally true could be to say something false or to say nothing. If we wish to maintain the parallel with the universal truths, we can take  $[T]$  to apply only to unworldly sentences to form an unworldly sentence. Otherwise, we can allow  $[T]$  to apply to both worldly and unworldly sentences to form an unworldly sentence. For the time being, we shall keep both options open, referring to the first as the narrow interpretation of  $[T]$  and the second as the broad interpretation. The notion of *formula* then is defined by the following clauses:

- (i) For every natural number  $i$ ,  $A_i$  and  $P_i$  are worldly and unworldly formulas, respectively;
- (ii) If  $A$  is a worldly formula then so is  $\neg A$ , and if  $P$  is an unworldly formula then so is  $\neg P$ ;

- (iii) If one or both of  $X$  and  $Y$  are worldly formulas then so are  $(X \wedge Y)$ ,  $(X \vee Y)$ ,  $(X \rightarrow Y)$  and  $(X \leftrightarrow Y)$ ; If they are both unworldly formulas then so are  $(X \Box Y)$ ,  $(X \Diamond Y)$ ,  $(X \rightarrow Y)$  and  $(X \leftrightarrow Y)$ ;
- (iv) If  $X$  is a formula (i.e., a worldly formula or an unworldly formula) then  $[N]X$  is an unworldly formula;
- (v) If  $A$  is a worldly formula then  $[U]A$  is an unworldly formula; and
- (vi) (narrow interpretation) If  $P$  is an unworldly formula then  $[T]P$  is an unworldly formula, or
- (vi') (broad interpretation) If  $X$  is a formula then  $[T]X$  is an unworldly formula

A model stipulates truth-values of unworldly sentence letters directly, and the truth-values of worldly sentences at each world. More precisely, we take a model to be a triple  $(W, w_0, V)$ , where  $W$  is a non-empty set (the set of possible worlds),  $w_0 \in W$  (the actual world), and  $V$  is a function (the valuation) that assigns a truth value to each unworldly sentence letter  $P$ , and a set of worlds  $S \subseteq W$  (the truth set) to each worldly sentence letter  $A$ . The base clause of the truth definition asserts that an unworldly sentence letter is true in a model if it is assigned truth by the valuation, and false if it is assigned falsity and that a worldly sentence letter is true in the model at a world  $w$  if  $w$  is an element of the truth set assigned to it by the valuation and false in  $w$  otherwise. The inductive clauses extend the application of *true* and *false* to arbitrary formulas preserving the condition that unworldly sentences are true or false in a model and worldly sentences are true or false in a model at a world. This will require several clauses for each connective.

In the case of conjunction, for example, we will need to specify that for a model  $M=(W, w_0, V)$  and a world  $w \in W$

$$M \models P \& Q \text{ iff } M \models P \text{ and } M \models Q;$$

$$(M, w) \models P \& A \text{ iff } M \models P \text{ and } (M, w) \models A;$$

$$(M, w) \models A \& P \text{ iff } (M, w) \models A \text{ and } M \models P;$$

$$(M, w) \models A \& B \text{ iff } (M, w) \models A \text{ and } (M, w) \models B.$$

For universal truth, the appropriate clause in the definition for truth in a model  $M=(W, w_0, V)$  is

$$M \models [U]A \text{ iff } (M, w) \models A \text{ for all } w \in W.$$

The necessity operator requires two clauses. Applied to a worldly formula, it indicates that the formula is universally true; applied to an unworldly formula, it indicates that the formula is true.

$$M \models [N]A \text{ iff } (M, w) \models A \text{ for all } w \in W;$$

$$M \models [N]P \text{ iff } M \models P.$$

The possibility operator is understood as the dual of the necessity operator in the usual way:  $\langle N \rangle A =_{\text{def}} \neg [N] \neg A$ . Notice that if a sentence is unworldly and negation is classical then necessity and possibility are both equivalent to truth, i.e., the principles  $[N]P \leftrightarrow P$  and  $\langle N \rangle P \leftrightarrow P$  are both valid.

All that remains is the operator for transcendental truth. On the narrow interpretation, we need only one clause

$$M \models [T]P \text{ iff } M \models P.$$

On the broad interpretation, we supplement the above clause with a second, equally simple one:

$$\text{Not } M \models [T]A.$$

To complete our semantics, we stipulate that  $A$  is true in  $M$  if it is true in the actual world of  $M$ , that  $X$  is valid if it is true in all models and that  $X$  and  $Y$  are logically equivalent if  $X \leftrightarrow Y$  is valid.

It follows that validity is closed under sort-preserving substitutions for sentence letters as well as under replacement of subformulas by logical equivalents of the same sort. Of course validity is also closed under tautological consequence.

With this bit of formal machinery in place, we might pause to ask what light, if any, it might shed on Fine's puzzle. There are two salient choices for representing the puzzle argument within our expanded modal language: taking the modalities to be about universal truth, and taking them to be about necessary truth

1.  $[U]P_1, \langle U \rangle A_1 \models \langle U \rangle (P_1 \ \& \ A_1)$ , and
2.  $[N]P_1, \langle N \rangle A_1 \models \langle N \rangle (P_1 \ \& \ A_1)$ ,

where  $P_1$  stands in for *Socrates is human* and  $A_1$  for *Socrates exists*. On the first choice, premise one is just ill-formed. We are interpreting *It is necessary that Socrates is human* in a way that is not just false, but incoherent. That seems implausible. On the second choice, the argument is sound and valid. Here we need some explanation of our initial reluctance to accept the conclusion. Fine suggests that it is part of a general reluctance to apply possibility to hybrid sentences. I am skeptical that we exhibit such a general reluctance. One explanation for our reluctance to accept that it is possible that Socrates is human and does not exist, might come from another of Fine's observations—that in considering this example we have a tendency to take *human* to mean *existing human*. If we succumb to this tendency we will think that the conjuncts inside the possibility operator are contraries and the assertion that their conjunction is possible is false. In that case, however, one would think that we would construe *human* the same way in premise one. *Socrates is human* would then be worldly and premise one would assert that it is universally true that Socrates is human. We are now faced with two choices. If universal truth is taken in the strong sense (true in all worlds) then premise one is false. If it is taken in the weak sense (true in all Socrates-containing worlds), premise two is false.

These considerations suggest that there *is* a simple puzzle (or a simplified formulation of Fine's puzzle), perhaps a little less dramatic, whose solution hinges exactly on the worldly/unworldly distinction. How is it that, on the ordinary, every-day sense of *necessarily*, *Socrates is necessarily human* is true while *Socrates necessarily exists* is false? If the ordinary, every-day sense of necessity is truth in all worlds then the first sentence is false. If it is truth in all worlds where the objects mentioned exist then the second sentence is true. The view of

necessity presented here provides a plausible explanation. Socrates is necessarily human because his being human is a transcendental truth, i.e., it is true regardless of world. He does not necessarily exist because he does not exist in all worlds, i.e., his existence is not a universal truth.

To say that different considerations underlie the necessity of different propositions is not to say that different senses of the word *necessarily* are used in the expression of these propositions. Even when applied to worldly sentences, which are necessary exactly when they are true in all worlds and perhaps *because* they are true in all worlds, *necessary* does not *mean* true in all worlds. The situation might be contrasted with its temporal analog. Consider the sentences *Socrates is always human* and *Socrates always exists*. Again the second sentence seems false. (Indeed he no longer exists now.) To say the first sentence is true, however, is not nearly so palatable as it was in the alethic case. It is much more natural to say here that, because *Socrates is human* is timeless, *Socrates is always human* is not a coherent sentence. The reason for the divergence, I think, is that whereas word *necessarily* does not mean *true in all worlds* the word *always* does mean *true at all times*.<sup>13</sup>

#### 4. Prior's System A and the logic of necessity, transcendence and universality

Once we have specified the formal language with the three special operators and provided it with a semantics it is natural to ask about the resulting logic. Let us call the language of worldly and unworldly formulas set forth in the previous section  $\mathcal{L}_{NTU}$  and set of formulas of  $\mathcal{L}_{NTU}$  that are valid according to our semantics  $\mathbf{L}_{NTU}$ .

Given the valid principle of substitution of logical equivalents, the following valid schemes allow us to eliminate occurrences of  $[N]$ :

(N1)  $[N]A \leftrightarrow [U]A$ ,

(N2)  $[N]P \leftrightarrow P$ .

Furthermore, on either the broad or the narrow interpretation,  $[T]$  can be similarly eliminated. The relevant biconditionals would either be

(T1)  $[T]P \leftrightarrow P$

alone, or that scheme together with

(T2)  $[T]A \leftrightarrow \Box$ .

Thus, to axiomatize  $\mathbf{L}_{NTU}$  it is sufficient to add a few simple biconditional schemes to the axioms for the logic of  $[U]$ . The logic of  $[U]$  is exactly what should correspond to Arthur Prior's System A.

Prior's avowed motivation in considering System A was to represent views according to which iterated modalities are meaningless.<sup>14</sup> Prior's "A-formulas" and "P-formulas" are just our

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<sup>13</sup>Note that for this reason the temporal analog of Fine's puzzle seems much less paradoxical. We are not so naturally inclined to accept the first premise, *it is always the case that Socrates is human*. If not false, that sentence seems inappropriate or ill-formed in a way that the alethic version does not.

<sup>14</sup> It is important to distinguish Prior's system from other rudimentary forms of modal logic that

worldly and unworldly formulas. He points out that in such a language we can express the following formulas, all of which are theorems of S5, but not of S4:

- 1)  $\Box(A \rightarrow \Box B) \rightarrow \Box(\Diamond A \rightarrow B)$ ,
- 2)  $\Box(\Diamond A \rightarrow B) \rightarrow \Box(A \rightarrow \Box B)$ ,
- 3)  $\Box(A \rightarrow \Box B) \rightarrow (\Diamond A \rightarrow \Box B)$ ,
- 4)  $\Box(\Diamond A \rightarrow B) \rightarrow (\Diamond A \rightarrow \Box B)$ .

The system A is (rather tentatively) identified as the logic obtained by adding the following two rules to propositional logic:

LA1. If  $\vdash (A \rightarrow X)$  then  $\vdash (\Box A \rightarrow X)$ ,

LA2. If  $\vdash (P \rightarrow A)$  then  $\vdash (P \rightarrow \Box A)$ .

$\Diamond$  is taken to be defined from  $\Box$  in the usual way and the system is taken to be closed under modus ponens which, in the presence of the theorems of propositional logic is equivalent to its being closed under tautological consequence. Because the rules are schematic, the theorems are also closed under substitution, but care must be taken that A-formulas and P-formulas are replaced only by others of the same sort.

Prior's confidence that this is the system for which he is searching seems to stem in large part from his ability to prove each of the four test formulas within it. His only semantic discussion of the system is the presentation of an ingeniously gerrymandered infinite-valued matrix that he plausibly conjectures to determine A. With the benefit of a half century or so of further development of modal logic, however, it is not difficult to show that A axiomatizes

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are similarly motivated. Prior himself (op cit p137) mentions a system he calls A' in which the necessity and possibility formulas cannot be applied to any formula that already contains such an operator. (So we can get the language of A' by taking the hybrid truth-functional combinations to be unworldly rather than worldly.) Although Prior was apparently unaware of it, a system like A' is already mentioned in [Parry 1953]. Parry identifies the logic S0.1 with the theorems of S2 of zero and first degree modality. This logic appears again as the "basic" modal logic B in [Pollock], which exhibits a complete axiomatization and a proof that it coincides with the zero and first degree theorems of KT and of all the other Lewis systems as well. In the long-gestating manuscript of Fine and Kuhn, S0.1 is treated as a kind of stepping-stone to the system S5.

In addition to A and A', Prior considers the possibility of a system A'', in which, using our terminology, all the boolean connectives except negation produce unworldly sentences as well as a system A''', in which necessity and possibility operators apply to worldly sentences to produce worldly sentences, but cannot apply to formulas whose construction has involved unworldly elements. So, for example, A'' admits as well-formed, the expression  $\neg \Box \neg A1$  but not  $\Box \neg \Box A1$  or  $\Box A1 \vee A2$ . A''' admits  $\Box \neg \Box A1$  but not  $\Box(A1 \vee P2)$ .

A' captures something like Fine's notion of unextended necessity. It does permit the formation of the boolean hybrids, but it does not allow necessity to be applied to such hybrids. None of the Prior systems admit  $\Box A1$  and  $\Box P1$ , while barring  $(A1 \vee P1)$  and while barring  $\Box A1 \vee P1$ , which would seem to be required for a system capturing extended necessity. (Superextended necessity, of course, is captured by the traditional syntax.)

universal truth according to the possible worlds semantics given above.

Let us first observe that we can replace Prior's LA1 with the T axiom,  $\Box A \rightarrow A$ , with no change to the resulting logic. Since  $\vdash A \rightarrow X$  and  $\vdash \Box A \rightarrow A$  together imply  $\vdash \Box A \rightarrow X$ , LA1 is a derived rule within the resulting system. Conversely, applying LA1 to the tautology  $A \rightarrow A$  gives us the T-axiom  $\vdash \Box A \rightarrow A$ .

Observe next that, by taking P in LA2 to be  $(P_1 \vee \neg P_1)$ , we derive the rule of necessitation.

In addition, we can prove the K axiom.

1.  $(A \rightarrow B) \rightarrow (A \rightarrow B)$  propositional logic
2.  $\Box(A \rightarrow B) \rightarrow (A \rightarrow B)$  1 LA1
3.  $A \rightarrow (\Box(A \rightarrow B) \rightarrow B)$  2 tautological consequence
4.  $\Box A \rightarrow (\Box(A \rightarrow B) \rightarrow B)$  3 LA1
5.  $\Box(A \rightarrow B) \wedge \Box A \rightarrow B$  4 tautological consequence
6.  $\Box(A \rightarrow B) \wedge \Box A \rightarrow \Box B$  5 LA2
7.  $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$  6 tautological consequence

A similar derivation establishes

$$(U) \quad \Box(P \rightarrow A) \rightarrow (P \rightarrow \Box A)$$

Since the rule LA2 is immediately derivable from U and necessitation, we have now established that Prior's system A can be axiomatized by axiom schemes T and U and the rules of necessitation and modus ponens. Since K is derivable in this system it can be identified, in the now standard notation, as KTU the smallest normal Kripke logic containing T and U.

Completeness of KTU for the intended semantics can be established by familiar methods. Take any KTU-consistent set  $\Gamma$ . We construct a model  $M=(W, w_0, V)$  that satisfies  $\Gamma$  as follows. Let  $w_0$  be a maximal KTU-consistent extension of  $\Gamma$ . Let  $W$  be the set of all KTU-consistent sets  $u$  of formulas with the properties that: (i) for all unworldly formulas  $P$ ,  $P \in w_0$  iff  $P \in u$  and (ii) for all worldly formulas  $A$ ,  $\Box A \in w_0$  implies  $A \in u$ . (The presence of the T-axiom ensures that  $W$  contains  $w_0$ .) For any worldly sentence letter  $A$  and any  $u \in W$ , let  $V(A, u) = T$  if  $A \in u$  and let  $V(A, u) = F$  otherwise. For any unworldly sentence letter  $P$ , let  $V(P) = T$  iff  $P \in w_0$ . We can prove by formula induction that the following two conditions obtain:

- (i)  $M \models P$  iff  $P \in w_0$  and
- (ii) for all  $u \in W$ ,  $(M, u) \models A$  iff  $A \in u$

Here, for example, is the case where  $P = \Box A$ .

If  $P \in w_0$ , then, by the definition of  $W$ ,  $A$  is a member of every set  $u$  in  $W$ . By induction hypothesis,  $(M, u) \models A$  for every  $u \in W$ . By the truth definition again,  $M \models \Box A$ , and so one direction of the equivalence has been shown.

For the other direction, suppose  $P \notin w_0$ . Let  $u^- = \{ Q : Q \in w_0 \} \cup \{ B : \Box B \in w_0 \} \cup \{ \neg A \}$ . Suppose, for reductio, that  $u^-$  is not consistent. Then  $\vdash (Q_1 \wedge \dots \wedge Q_n) \wedge (B_1 \wedge \dots \wedge B_n) \wedge \neg A \rightarrow \perp$ , for appropriate  $Q_i \in w_0$  and  $B_j$  such that  $\Box B_j \in w_0$ . By tautological consequence,  $\vdash (Q_1 \wedge \dots \wedge Q_n) \rightarrow ((B_1 \wedge \dots \wedge B_n) \rightarrow A)$ . By (U),  $\vdash (Q_1 \wedge \dots \wedge Q_n) \rightarrow \Box((B_1 \wedge \dots \wedge B_n) \rightarrow A)$ . Since each  $Q_i$  is in  $w_0$  and  $w_0$  is maximal KTU-consistent, the antecedent of this formula is in  $w_0$ , and therefore  $\Box((B_1 \wedge \dots \wedge B_n) \rightarrow A)$  is in  $w_0$  as well. By principles of K, this implies  $\Box B_1 \wedge \dots \wedge \Box B_n \rightarrow \Box A$  is in  $w_0$ . Since the  $B_j$ 's were chosen so that their necessitations were in  $w_0$ , this implies that  $\Box A \in w_0$ , which contradicts the initial supposition. Hence  $u^-$  is KTU-consistent and can be extended to a maximal KTU-consistent set  $u$ , that meets the conditions for membership in  $W$ . Since  $\neg A \in u$  and  $u$  is consistent,  $A \notin u$ . By induction hypothesis,  $(M, u) \models A$ , and so by the truth clause for  $\Box$ ,  $(M, w_0) \models P$ , as was to be shown.

Thus we have established that KTU is the logic of  $[U]$  and that the logic  $\mathbf{L}_{\text{NTU}}$  of  $[N], [T]$  and  $[U]$  is obtained by adding a few biconditionals to this. Two alternative axiomatizations of  $\mathbf{L}_{\text{NTU}}$  are worth noting. The first (which was pointed out to me by Frabrice Correa) replaces U by

$$(C) \quad P \rightarrow \Box(A \rightarrow P).$$

Here is a sketch of the derivations showing the equivalence of C and U.

1.  $P \rightarrow (A \rightarrow P)$  propositional logic
2.  $\Box(P \rightarrow (A \rightarrow P))$  1 necessitation
3.  $\Box(P \rightarrow (A \rightarrow P)) \rightarrow (P \rightarrow \Box(A \rightarrow P))$  U
4.  $P \rightarrow \Box(A \rightarrow P)$  2,3 tautological consequence

1.  $(P \rightarrow A) \rightarrow ((B \rightarrow B) \rightarrow A)$  propositional logic
2.  $\Box(P \rightarrow A) \rightarrow \Box((B \rightarrow B) \rightarrow A)$  1, necessitation + K + tautological consequence
3.  $\Box(((B \rightarrow B) \rightarrow P) \rightarrow A) \rightarrow (\Box((B \rightarrow B) \rightarrow P) \rightarrow \Box A)$  K
4.  $P \rightarrow \Box((B \rightarrow B) \rightarrow P)$  C
5.  $\Box(P \rightarrow A) \rightarrow (P \rightarrow \Box A)$  2,3,4 tautological consequence

To obtain the second alternative, note that we could view the schema N1 as a device for eliminating  $[U]$  in favor of  $[N]$  rather than the reverse. Thus to axiomatize  $\mathbf{L}_{\text{NTU}}$  it would be sufficient to add a few biconditionals to the logic of  $[N]$ , and we might expect that logic to be just the familiar S5. A proof that the logic of  $[N]$  is indeed S5 can be given in two parts. First, one checks that every instance of an S5 axiom is provable and that necessitation (i.e., prefixing with  $[N]$ ) preserves provability. For example, a check of the K axiom requires showing that the schema  $[N](X \rightarrow Y) \rightarrow ([N]X \rightarrow [N]Y)$  is provable in each of the four cases where  $X$  and  $Y$  are worldly or unworldly. That establishes that the fragment contains at least S5. To show that it cannot contain any more, we can use the fact that the only normal extensions of S5 contain some “domain-size” schema  $\Diamond X_1 \wedge \dots \wedge \Diamond X_n \rightarrow \bigvee_{i < j < n} \{\Diamond(X_i \wedge X_j)\}$ . Since no such schema is valid for  $\Box=[N]$  in our semantics the desired result follows.

KTU and KTC are both simpler than S5 in that they require no axioms with nested modalities. Taking the connective that applies to all formulas as primitive may be seen as having

some theoretical advantage, but one pays a price in complexity of axioms.<sup>15</sup>

## 5. Actuality

The interpretation of the language  $\mathcal{L}_{NTU}$  and logic  $\mathbf{L}_{NTU}$  as appropriately reflecting the worldly/unworldly distinction trades on the idea that a worldly sentence can be viewed as having an implicit world-variable that becomes bound when certain operators apply to the sentence. An unworldly sentence can be viewed as having no such implicit variable. Fine's initial characterization of the distinction, however, was in terms of truth-makers. A worldly truth is made true by circumstances in the world. An unworldly truth is not made true by any such circumstances. It is not obvious that these two characterizations coincide. One might be especially wary if one notes that becoming bound by a quantifier is not the only way by which a free-variable can be removed from a sentence. The variable can also be *instantiated* with a proper name. Consider again the analogy with time. If the sentence *Hillary reaches the summit* is viewed as having a temporal free variable, then in *Hillary reached the summit at noon (today)* that free variable is no longer present. Nevertheless, under a seemingly ordinary conception of "makes true", *Hillary reached the summit at noon* is made true by circumstances that obtained at noon (and perhaps those that obtained at some prior moments as well). On this ordinary conception, then, we might deny that *Hillary reached the summit at noon* is timeless. Its truth does not depend on *current* circumstances, of course. There is nothing that can be done now to fix the truth value of *Hillary reached the summit at noon*. We might call sentences like this that describe how things are independently of present circumstances *temporally immutable*. It may seem reasonable to stretch the notion of *timeless* to include the temporally immutable. When the time referred to happens to coincide with present moment, however, the stretch seems more difficult. If *Hillary reaches the summit at noon* is made true by circumstances at noon, then at noon that sentence is made true by the circumstances then obtaining. What, one might think, could be more timely than that?

If the truth-maker characterization of the timely/timeless distinction is to deliver the verdicts delivered by the characterization in terms of free-variables, the idea that *Hillary reaches the summit at noon* is made true by circumstances then obtaining must be resisted. "Circumstances" should not be viewed as themselves containing times. The circumstances at noon might include Hillary's being at the summit and his having recently been moving towards the summit. They should not (contrary to some ordinary usage) be seen as including its being noon. On this understanding, the circumstances might make it true that Hillary reaches the summit, but they can never (even at noon) make it true that he reaches the summit at noon.<sup>16</sup>

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<sup>15</sup> It should also be noted that extending the formal systems described here to predicate logic would face some conceptual challenges. In addition to the issue raised in note of whether predicates have stable signatures, there is a similar issue concerning quantification. *Five is less than seven* and *there is something less than seven* are both unworldly, so one might think that existential quantification takes us from the unworldly to the unworldly. But *Socrates is human* is alleged to be unworldly, while *there is something human* seems to be worldly.

<sup>16</sup> The circumstances could, of course, include the sun's being directly overhead, and the

The two ways of drawing the timeless/timeless boundary can also be seen using Fine's divine power metaphor. On one understanding, God could make it the case that Hillary reached the summit at noon in either of two ways. He could appropriately change what Hillary does at noon and suitable preceding times while leaving the structure of time unchanged or, alternatively, if Hillary has reached the summit at some other time, He can make that time noon, while leaving Hillary's actions unchanged. On another understanding, however, God's powers consist only in the ability to choose a possible world as actual and a time as present. If it is possible that Hillary reaches the summit then, by making a world where he does so actual, God can make it happen that Hillary reaches the summit. He cannot, however, by choosing a time within that world as present, make *Hillary reaches the summit at t* for some particular time *t*, true. The truth of *Hillary reaches the summit at t* is independent of the time that is present. It is this second conception of more limited divine power, that must be invoked to get the distinction Fine is apparently after.

What has been said in this section about times applies equally to worlds. On the understanding that concerns us, *Socrates exists* is made true or false by the circumstances in *w*, but *Socrates exists in w* is not. By choosing this world as actual, God may have made the sentence *Socrates exists* true, but He did not thereby make the sentence *Socrates exists in w* even if 'w' names the actual world. The existence of Socrates in *w* is independent of which world is actual.

While reference to particular times seems common in ordinary language,<sup>17</sup> reference to particular possible worlds does not. There is a view, however, on which certain uses of the expression *actually* do provide means for referring to a particular possible world, namely the actual one. These *logical* uses of the term are to be distinguished from *rhetorical* uses in which it may serve merely to acknowledge that the audience may find the attached proposition doubtful or surprising (as in *Nixon actually went to China*). To capture the logic of the logical *actually* we may consider adding an actuality operator, [A], to familiar systems of alethic modal logic. Here we are considering the view that *it is actually the case that S* says that *S* is true at a particular world and that the circumstances in a world do not make it true or false that any proposition is true in that world (though they do make it true that such propositions are true). Under these

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inclusion of this circumstance might seem to make it true that he reached the summit at noon after all. In the example, however, *noon* was supposed to be a term referring directly to a particular time. One might question whether there are such terms in natural language on grounds that, for example, calendar dates refer to numbers of planetary revolutions since the time at which a particular historical event was believed by calendar-makers to have occurred. (But would this really not be 2012 if it were discovered that Pope Gregory XIII and his astronomer-advisor Christopher Clavius actually knew that Jesus had been born a few years earlier than their calendar indicated or that the earth had orbited the sun twice in some period that the history books describe as a single year?) The theoretical point, that the proposition that an event occurs at a particular time must be timeless, would remain true, however, even if that proposition cannot be expressed.

<sup>17</sup>See previous footnote about possible grounds to doubt this claim.

circumstances then, we should think about adding to the language  $\mathcal{L}_{NTU}$ , an operator  $[A]$  of the same syntactic category as  $[U]$ , i.e., it applies to worldly sentences to form unworldly ones. The scheme  $[A]A \rightarrow \Box[A]A$ , which exercised pioneers in the logic of *actually* can then be understood as having two correlates in our sorted language.  $[A]A \rightarrow [U][A]A$ , which is ungrammatical, and  $[A]A \rightarrow [N][A]A$ , which is valid only because the application of  $[N]$  to unworldly sentences merely indicates that they are true.

This picture fits nicely with a metaphysical view that is closely associated with Fine. That view is *modal actualism*, according to both modality and actuality are to be taken seriously. Applied to possible worlds, it maintains that the actual world has special ontological status; talk of other possible worlds is intelligible and useful, but must ultimately be understood in other terms<sup>18</sup> (which, for Fine, may include *necessary* and *possible*). A contrary view, often associated with David Lewis,<sup>19</sup> holds that the actual world is one among many possible worlds, each of which is as actual to its inhabitants as we are to our own. Just as I may think that the person to whom I refer by the word *I* is special without fully appreciating that he is special only to me, we may think in this world that the world I call *actual* is special without fully appreciating that it is special only here. This second, non-actualist, view fits more naturally with a view that  $[A]$  should be of the same syntactic category as negation, applying to worldly sentences to form worldly sentences. Indeed, one might think that on this view  $[A]A$  should be true in exactly the worlds where  $A$  is true, making  $[U](A \leftrightarrow [A]A)$  valid. But in that case  $[A]A$  and  $A$  would be interchangeable in any formula and the logic of actuality would seem to be of little interest. Lewis and others point out, however, that, when other possible worlds are under consideration, the word *actually* may be understood relative to *this* world (its “primary” sense) or those other worlds (its “secondary”) sense. As examples, Lewis points to

If Max ate less he would be thinner than he actually is, (primary) and

If Max ate less he would actually enjoy himself more. (secondary).<sup>20</sup>

This distinction opens the door for an interesting logic of actuality for the non-actualist. Indeed, if we restrict ourselves to the primary sense of *actually* and to the semantics of languages as spoken in this possible world (*our* English?), then it would seem that the logic of *actually* should be essentially the same for the actualist and the non-actualist.  $[A]A$  may be worldly in a technical sense, but if it is only used in this world and always given the primary interpretation, then we can safely treat it as unworldly.

The addition of  $[A]$  to a language for modal logic raises at least one other philosophical question. In the standard (normal) modal logics, if a formula is true at the actual world in all models, it is true at all worlds in all models. (For if it were false at any  $w$  in some model, then it would be false at the actual world in the model where that  $w$  is the actual world.) So the notion

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<sup>18</sup>This view (described here as the application of modal actualism to possible worlds) is discussed under the label *soft actualism* in [Adams].

<sup>19</sup>See [Lewis]. Lewis labels this the *indexical* view, but this invites confusion between the metaphysical view about actuality and a corresponding semantic view about the expression *actually*.

<sup>20</sup>[Lewis] p 185.

of truth in all models coincides with the notion of truth at all worlds in all models. With the addition of the actuality operator, this coincidence fails.  $[A]A_1 \leftrightarrow A_1$  is true in all models but it is false at  $w$  in any model in which  $A_1$  is assigned different truth values in  $w$  and the actual world. So there is a question as to whether validity, which we intend to capture a notion of logical truth, should be defined as truth in all models or truth at all worlds in all models.<sup>21</sup> The first option initially seems the natural way to extend our Tarskian paradigm. Choosing it, however, will lead us to accept that there are logical truths (like  $[A]A \leftrightarrow A$ ) that are merely contingent, which seems contrary to long-held understanding of the nature of logic. This issue has been the subject of controversy in the recent philosophical literature.<sup>22</sup> I am not sure the extent to which the issue is substantive rather than terminological. We might take logical truth to be truth in virtue of logical form or necessary truth in virtue of logical form. Either definition would seem to be consistent with the traditional understanding (perhaps because it has not been appreciated that the two come apart). The first option, however does make technical investigation a little more difficult.<sup>23</sup> If validity is defined as truth in all models then two rules characteristic of standard modal logics are no longer validity-preserving: necessitation ( $[A]A \leftrightarrow A$  is valid but  $[N]([A]A \leftrightarrow A)$  is not) and replacement, (although  $[A]A$  is equivalent to  $A$ ,  $[N](A \leftrightarrow A)$  is valid while  $[N]([A]A \leftrightarrow A)$  is not). For this reason, I choose the second option in the discussion that follows.

Let  $\mathcal{L}_{NTUA}$  be the language obtained by adding to  $\mathcal{L}_{NTU}$  the unary operator  $[A]$  which applies only to worldly formulas to produce unworldly ones. Add to the truth definition for  $\mathcal{L}_{NTU}$  a clause stating that, for any model  $M=(W, w_0, V)$ ,  $M \models [A]A$  iff  $M \models A$ , and call a formula valid if it is either unworldly and true in every model, or worldly and true at all worlds in every model. Finally, let  $L_{NTUA}$  be the set of valid formulas of  $\mathcal{L}_{NTUA}$ . The paper concludes with a simple axiomatization of  $L_{NTUA}$ .

As before, we may take as axioms biconditionals allowing us to eliminate  $[T]$  and  $[N]$  in favor of  $[U]$ , and now  $[A]$  as well. Let  $KTU-A$  be the result of adding to  $KTU$ , the following axiom and rule schemes.

$(K_{[A]})$	$[A](A \rightarrow B) \rightarrow ([A]A \rightarrow [A]B)$
$(U_{[A]})$	$[A](P \rightarrow A) \rightarrow (P \rightarrow [A]A)$
$(Neg_{[A]})$	$[A]\neg A \rightarrow \neg [A]A$
$(Act)$	$\vdash A \text{ implies } \vdash [A]A$

As with  $KTU$ , it can be shown that  $KTU-A$  is closed under tautological consequence and replacement of provable equivalents of the same sort. Note also that  $Neg_{[A]}$  can be strengthened to a biconditional. For  $[A]\neg\neg A \rightarrow \neg\neg [A]A$  is an instance of  $Neg_{[A]}$ . By contraposition, this implies  $\vdash [A]\neg A \rightarrow [A]\neg\neg A$ . Replacing  $\neg\neg A$  by  $A$  implies  $\vdash [A]\neg A \rightarrow A$ , as

<sup>21</sup>In [Crossley and Humberstone] this issue is discussed using the labels “real-world” validity and “general” validity. Analogous issues arise in logics for indexicals. See [Kaplan] and [Kamp], both of which choose what is referred to above as option one. It is not obvious that the same considerations should apply in the cases of *now* and *here* on the one hand and *actually* on the other.

<sup>22</sup>See, for example, [Zalta].

<sup>23</sup>Not impossibly so, however. See [Hazen].

claimed. We will let  $\text{Neg}'_{[A]}$  be the converse to  $\text{Neg}_{[A]}$ .

Completeness of KTU-A for the intended semantics can now be proved by methods similar to those employed above for KTU. Take a KTU-A-consistent set  $\Gamma$ . Extend it to a maximal KTU-consistent set  $\Gamma^*$ . As before, this set can be regarded as providing, both a world where the worldly sentences of  $\Gamma$  are true, and information about the unworldly sentences true in the model to be constructed. In this case, however, it cannot be regarded as representing the actual-world of the model. That job, we assign to the set  $[A](\Gamma^*) = \{Q : Q \in \Gamma^* \} \cup \{A[A]A \in \Gamma^*\}$ .

Claim:  $[A](\Gamma^*)$  is maximal KTU-A-consistent.

Proof.

First, prove that  $[A](\Gamma^*)$  is KTU-A-consistent. Suppose otherwise. Then  $\vdash Q \rightarrow (A \rightarrow \perp)$ . By the rule Act,  $\vdash [A](Q \rightarrow (A \rightarrow \perp))$ . By  $U_{[A]}$  and tautological consequence,  $\vdash Q \rightarrow [A](A \rightarrow \perp)$ , where, we may assume,  $\perp$  is a worldly contradiction  $B \wedge \neg B$ . Since  $Q \in \Gamma^*$ ,  $[A](A \rightarrow \perp) \in \Gamma^*$ . By  $K_{[A]}$  and tautological consequence,  $[A]\perp \in \Gamma^*$ . Substituting tautological equivalents,  $[A]\neg \top \in \Gamma^*$ . By  $\text{Neg}_{[A]}$ ,  $\neg [A]\top \in \Gamma^*$ . But also, by rule Act,  $[A]\top \in \Gamma^*$ , which would violate the consistency of  $\Gamma^*$ . So  $[A](\Gamma^*)$  is consistent, as was to be proved.

Next, prove that  $[A](\Gamma^*)$  is maximal. Suppose  $X \notin [A](\Gamma^*)$ . There are two cases. If  $X$  is unworldly then, by definition, it can't be in  $\Gamma^*$ . Since  $\Gamma^*$  is maximal,  $\neg X \in \Gamma^*$  and so by definition  $\neg X \in [A](\Gamma^*)$ , as required. If  $X$  is worldly, then by definition of  $[A](\Gamma^*)$ ,  $[A]X \notin \Gamma^*$ , and so  $\neg [A]X \in \Gamma^*$ . By  $\text{Neg}'_{[A]}$ ,  $[A]\neg X \in \Gamma^*$ , and, by definition of  $[A](\Gamma^*)$ ,  $\neg X \in [A](\Gamma^*)$ , as required.

Now we construct our model for  $\Gamma$  as before with  $[A](\Gamma^*)$  playing the role that  $\Gamma^*$  had played before. Let  $w_0 = [A](\Gamma^*)$  and let  $W$  be the set of all maximal consistent sets  $u$  that contain the same unworldly formulas as  $w_0$  as well as every worldly formula whose necessitation is in  $w_0$ . Since our logic contains the T-axiom, it follows that  $w_0 \in W$ .

It remains only to verify the truth lemma. The  $\Box$  case is as before. The  $[A]$  case requires that we prove  $M \models [A]A$  iff  $[A]A \in w_0$ . Suppose first that  $M \models [A]A$  but not  $[A]A \in w_0$ . Then  $\neg [A]A \in w_0$ . By  $\text{Neg}'_{[A]}$ ,  $[A]\neg A \in w_0$ . Since  $[A]\neg A$  is unworldly, it is also in  $\Gamma^*$ . By the definition of  $w_0$  then,  $\neg A \in w_0$ , and so not  $A \in w_0$ . By induction hypothesis,  $(M, w_0) \not\models A$ , and so by truth definition,  $M \not\models [A]A$ . This contradicts our initial supposition and so this direction is proved. Suppose next that  $[A]A \in w_0$ . Since  $[A]A$  is unworldly, then it is also in  $\Gamma^*$ . By definition of  $w_0$ ,  $A \in w_0$ . By induction hypothesis,  $(M, w_0) \models A$ . By truth definition  $M \models A$ , as required.

It follows from the truth-lemma that  $(M, \Gamma^*) \models \Gamma$

At first glance our logic KTU-A appears to lack correlates of several of the characteristic axioms of Crossley and Humberstone's S5A. Conspicuously missing are axioms resembling the following (with notation slightly changed to better conform to ours):

$$(A1) [A]([A]p \rightarrow p),$$

- (A4)  $\Box p \rightarrow [A]p$ ,  
 (A5)  $[A]p \rightarrow \Box [A]p$ .

Within the sorted framework, the  $\Box$  in A5 can only be understood as  $[N]$ , and on this understanding A5 is provable from our N2. The other two axioms require a little more thought. Below are sketches of derivations establishing correlates of A4 and A1.

1.  $[U]A \rightarrow A$  T
2.  $[A]([U]A \rightarrow A)$  1 Act
3.  $[U]A \rightarrow [A]A$  2,  $U_{[A]}$  tautological consequence
  
1.  $\neg[A]A \rightarrow ([A]A \rightarrow A)$  tautology
2.  $[A](\neg[A]A \rightarrow ([A]A \rightarrow A))$  1 Act
3.  $\neg[A]A \rightarrow [A]([A]A \rightarrow A)$  2,  $U_{[A]}$  tautological consequence
4.  $A \rightarrow ([A]A \rightarrow A)$  tautology
5.  $[A](A \rightarrow ([A]A \rightarrow A))$  4 Act
6.  $[A]A \rightarrow [A]([A]A \rightarrow A)$  5,  $K_{[A]}$  tautological consequence
7.  $[A]([A]A \rightarrow A)$  3,6 tautological consequence

So, on closer examination, it turns out that the sorted system does have theorems corresponding closely to the axioms of S5A. The sorted formulation again seems to allow for simpler axioms (and a simpler completeness proof) than the more standard one.

## 5. Conclusion

Let us briefly review Fine's thoughts on necessity and transcendence and the emendations and refinements proposed here. Sentences that ascribe to an individual a property that it has by its very nature, according to Fine, are unworldly, and the propositions that they express are transcendental. Such propositions are necessary or possible only in an extended sense and, when these sentences are combined with ordinary worldly ones, the results are necessary or possible only in a superextended sense. This explains why someone who accepts the necessity of Socrates being human and the possibility of his not existing is still likely to balk at the possibility of his being human and failing to exist. I have argued that the ordinary, every-day sense of necessity encompasses Fine's extended and superextended necessities as well as those propositions true in all worlds. This undermines the idea that Fine's puzzle provides evidence for the worldly/unworldly distinction. I have suggested, however, that a slightly different puzzle (or a slightly different version of Fine's puzzle) does provide such evidence: why do we ordinarily judge *Socrates is human* to be necessary, but *Socrates exists* to be contingent? A plausible explanation is that sentences like *Socrates is human* are both necessarily true and true regardless of worldly circumstances. Even if this line of thinking does not convince us about sentences like *Socrates is human* it is still reasonable to think that other kinds of sentences should fit those two descriptions. One such comprises sentences expressing mathematical truths. Another comprises sentences that truthfully state that a sentence is *universally* true, i.e., true in every worldly circumstance and those that state that a sentence is true in some worldly circumstances.

Somewhat surprisingly, perhaps, this category is naturally understood to include sentences that truthfully state that a sentence is true in particular worldly circumstances, for example those circumstances that obtain in this, the actual, world.

A simple formal system that reflects and elucidates the worldly/unworldly and the necessary/universal distinctions has formulas of two sorts. Models determine truth values of worldly formulas at possible worlds; they determine truth values of unworldly formulas *simpliciter*. The logic of the operator  $[U]$ , for truth in all worlds, is exactly what Arthur Prior once sought under the name *System A*. Prior's conjectured axiomatization, it turns out, is correct. It is equivalent to a simpler one obtained by adding to a two-sorted version of KT the axiom schema  $\Box(P \rightarrow A) \rightarrow (P \rightarrow \Box A)$  (where A and P are worldly and unworldly formulas). To this we can add an actuality operator that accords with our understanding that sentences asserting the truth of worldly sentences at particular worlds are themselves unworldly. The logic that results is again shown to have a pleasantly simple axiomatization.

The argument that Fine's puzzle, as originally formulated, provides evidence that Socrates' humanness is a transcendental, rather than a worldly, fact assumes that we are more reluctant to attribute necessity to propositions true because of the nature of individuals than to propositions true because of worldly circumstances, and that we find it still more difficult to attribute necessity to "hybrid" sentences. A simpler version of the argument, however, reaches the same conclusion under a more plausible understanding of the relation between necessary and transcendental truth. We may question Fine's emphasis on his two-premise puzzle and even, perhaps, his principal example of a transcendental truth. It is difficult to deny, however, the interest and importance of the worldly/unworldly distinction and Fine's thought about it. We should be grateful to him for bring this, as many other neglected topics in metaphysics, into focus.

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