Humberstone, Lloyd, *Philosophical Applications of Modal Logic*, Milton Keynes: College Publications. 2016, pp vi + 578, US$33.61 (paperback).

This is a remarkable piece of work—a detailed and imaginative survey of topics in propositional modal logic, broadly construed, that contains new conceptual insights and technical developments on every topic surveyed. It purports to be a textbook, aiming to "convey some of the interest and charm of modal logic, and to put a reader new to the subject in a position to have an informed opinion as to its applicability to each of several areas of philosophical concern (belief, knowledge, obligation...) in which the merits of a 'modal' approach have been controversial." In truth, the book will probably be of at least as much interest to the teachers of such novice readers as to the neophytes themselves. It contains lucid exposition, technical elaboration, and commentary on topics that have interested the author over the course of a career approaching fifty years.

The first third of the book, intended to provide background for the philosophical applications to come, leaves the modal □ largely uninterpreted. The remainder comprises long surveys of deontic and epistemic/doxastic logic, and shorter sections on tense logic, "nomic" necessity and the author's own logic of "coming about." There is an appendix on natural deduction systems for modal logic, and a comprehensive bibliography that is itself worthy of notice. Even within the "background" portion, the choice of topics reflects the author's interests rather than following canons of the field, and the presentation is infused with original and interesting material. The size and scope of the book precludes more than a very cursory overview here. I will summarize some of the major themes and identify a few innovations within each. Deeper commentary on some specific points in the text is provided in a separate paper. (See Kuhn [2017].)

It might be best to begin at the end, with the remarkable appendix. One of the book's great virtues is in collecting, distilling and organizing so much of the mass of research that has appeared in disparate, sometimes obscure places. The last eminent chroniclers of record for philosophically-oriented modal logic, Hughes and Cresswell, produced a book with a bibliography of thirteen pages. Humberstone's has fifty-eight! Although the text focuses on particular issues, the reader is informed about topics omitted or slighted and directed to relevant sources in the bibliography. In such a rapidly expanding field, pursued by scholars from different disciplines and different continents, this provides an invaluable service. The reader learns of a number of cases already in which authors ignorant of each other's work have duplicated it. She is forewarned of logical and conceptual (and typographical!) errors she would encounter were she to consult the references. Of somewhat narrower concern, the bibliography and pointers to it within the text provide a useful guide to the author's own thought—some 60 of the 1200 references are to his own publications.

The background material, occupying the book's first two chapters, emphasizes monomodal normal logics with Kripke-style semantics. Among topics conspicuously omitted are quantification, "filtration" of models, and bi-simulation. The first two omissions are sensible, given the author's aims for the book, but the third is more surprising, since the idea is more general and scarcely more difficult than the notion of p-morphism employed in its place. The normal logics are nicely situated within a more general framework and a number of lesser-known alternative semantics for weaker logics are explored. Particular attention is

devoted to the investigation of modal *rules*, an undertaking that made a brief earlier appearance in the Hughes and Cresswell work cited above and that is considerably more difficult than the usual investigation of axioms. Another topic that gets unusual emphasis in the beginning chapters, and proves useful in later ones, concerns logical *embeddings*. The highlight is an informative proof of an old result of Kazuo Matsumoto that the mapping that takes A into □◊□A faithfully embeds S5 into S4 and a characterization of the range of logics that can play the role of S4 here. These ideas are relevant to another topic of recent interest emphasized here: the logics of complex modalities, in this case of □◊□.

The author takes great care to identify and credit the sources of the ideas presented. Perhaps out of modesty, however, he does not take similar care to identify what is new to the book. One obvious novelty is his attempt to take the little diagrams that commonly illustrate conditions on Kripke frames seriously as objects of study. This leads to an exploration of *alio-* forms of conditions on binary relations, obtained by restricting universal quantifiers with inequalities. For example, both $\forall x \forall y \forall z(Rxy \wedge Ryz \wedge x{\neq}y \rightarrow Rxz)$ and $\forall x \forall y \forall z(Rxy \wedge Ryz \wedge x{\neq}z \rightarrow Rxz)$ are considered conditions of alio-transitivity. Frequently (as in the first case above) an alio- form is logically equivalent to its non-alio parent. Sometimes, however, it is logically distinct, and may play a useful role as a condition on Kripke frames. A novelty somewhat similar in spirit is an inquiry into the relation between general and "piecewise" conditions on Kripke frames. The formulas $\forall x \forall y Rxy$ and $\forall w \forall x \forall y(Rwx \wedge Rwy \rightarrow Rxy)$, for example, express that R is universal and piecewise universal and correspond to distinct modal formulas. Yet another apparent innovation is the author's preferred method of showing that a class of frames is *modally undefinable* (i.e., that no formula characterizing it corresponds to a modal formula), by identifying a logic whose canonical frame lies within the class while other frames for it lie outside. This seems simpler than methods commonly employed.

The *applied* modal logics taken up in later chapters are classified according to whether the □-connective expresses a propositional attitude and whether it is veridical (i.e., conforming to the T-axiom, □A→A), although the book's organization does not quite fit the classification. Logics of nomic necessity (sections 2 and 3 of chapter 4) and coming about (chapter 6) are non-attidudinal and veridical. Deontic (sections 4, 5, and 8 of chapter 4) and tense (chapter 3) logics are non-attitudinal and non-veridical. Doxastic and epistemic logics (chapter 5) are attidudinal and non-veridical and attitudinal and veridical, respectively, although as Humberstone demonstrates, there is much to be gained by considering logics containing the □-connectives of both sorts. As reasonable as these taxonomic judgements are, many, as the author notes, have been contested.

The short chapter on tense logic is of mainly technical interest, lacking, for example, consideration of philosophical questions about time and determinism, and semantical questions about tenses in natural language. One highlight is a discussion of density and discreteness in orderings lacking linearity. The notions of having immediate successors and having successors with immediate predecessors come apart in this context. Another is the clever axiomatization of the tense-logical analog of the modal logic **K** by a single (two-way) rule: ⊢A∨*G*B iff ⊢*H*A ∨ B. As a corollary, we get a similar axiomatization of the monomodal logic KB with the rule: ⊢A∨□B iff ⊢□A∨B.

Suppose the worlds nomically accessible to a world w are those in which every sentence that is a law in w is true, and every world is nomically accessible to a lawless world.

Then every world is (nomically) accessible to world to which all worlds are accessible. It follows that every world can be reached in two steps. If logical possibility is truth at some world and nomic possibility is truth at some nomically accessible world, then A is logically possible if and only if it is nomically possible that it is nomically possible. Thus logical possibility is reducible in some sense to nomic. This contrasts with the more common idea that there is a reduction in the reverse direction: A is nomically possible if and only if it is logically compatible with the laws of nature. Something like this suggestion has been given a sustained defense by John Bigelow and Robert Pargetter. In one section on nomic necessity here, the author clears up a number of confusions in this discussion and confirms that the axioms suggested by Bigelow and Pargetter do the job for which they were intended. In another, he similarly polishes and refines John Bacon's axiomatization of "purely physical" (i.e., nomic but not logical) necessity.

The deontic logic portion of the book begins with a characteristically wide-ranging overview and guide to the literature. Humberstone is well aware of the many issues that arise from attempts to fit obligation, permission and prohibition within modal frameworks designed for other applications. He takes them as challenges to be overcome and occasions for technical exploration rather than refutations of the enterprise. Particular attention is paid to suggestions that obligation might be reduced to necessity, perhaps with the addition of a special normative sentential constant. In a formulation by Alan Ross Anderson, we have $\circ A = \Box(\neg A \rightarrow S)$, where S is a sentential constant saying that "the sanction is applied". In (logically tantamount) formulations of Timothy Smiley and Stig Kanger we have $\circ A = \Box(Q \rightarrow A)$, where Q expresses the laws relevant to the kind of obligation expressed by $\circ$. When the logic of the underlying necessity operator is the normal extension of KT by $\Diamond Q$ (or $\Diamond \neg S$ if we are following Smiley), the logic of the resulting obligation operator turns out to be the normal extension of KD by $O(O \rightarrow A)$, one of the more plausible candidate logics of obligation. Less credible reductions would identify $\circ$ with $\Diamond \Box$ (in which case the logic S4.2 for $\Box$ induces for $\circ$ the logic KD45, aka "the deontic S5"), or with $\Box \Diamond$ (in which case, as Humberstone demonstrates, no plausible logic for $\circ$ is likely to emerge). Technical explorations here pay dividends in later chapters, where the epistemic and doxastic interpretations of the operators lend more sense to the formulas investigated. The absence of the T-axiom $\Box A \rightarrow A$ from a plausible deontic logic provides Humberstone an excuse to include in this section a long rumination on the interesting general idea of "fully modalized" logics, in which non-trivial connections between modal and non-modal formulas are absent. A casual observation of Arthur Prior's provides another excuse for technical inquiry. Prior notes that truth is a logical intermediary between necessity and possibility in much the same way as singular statements are intermediaries between universal and existential ones. But in the deontic case there seems to be no comparable intermediary between what is necessary and what is permitted. One suspects that Prior's point may have had more to do with our everyday concepts rather than what we might logically construct from them. Nevertheless, the remark provides Humberstone the impetus for an interesting investigation into conditions under which the Lindenbaum Algebra of a modal logic is dense.

The chapter on epistemic and doxastic logic follows a familiar pattern. Areas of inquiry to be omitted are briefly explained and the reader is referred to relevant literature for more detail. In this case, omissions include some burgeoning areas of research that mark significant departures from the logical machinery under consideration in the book: multiple agents, public announcement, belief change, autoepistemic logic. As before, the author is well aware of issues raised by treatments within the older logical framework. He includes

extended discussion of several these, under the labels "logical omniscience," "introspection," and "concept possession." The first refers to the worry that standard epistemic logics may wrongly imply that all logical truths and all logical consequences of known truths are known; the second, to the worry that such logics may wrongly imply that what is known (or unknown) is always known to be so; the third to the worry that they may wrongly attribute knowledge of something to subjects with no grasp of concepts required to understand it. The formula $\Diamond\Box A$, identified earlier as an implausible candidate to express obligation, is much more plausible as an express belief when $\Diamond$ and $\Box$ are given epistemic interpretations: an agent believes A just in case he might, for all he knows, know it. Extended exploration of this idea leads the author to suggest, following Wolfgang Lenzen, that the correct epistemic logic (if it is anything of the kind his book concerns) lies between S4.2 (characterized by the axiom $\Diamond\Box A\rightarrow\Box\Diamond A$) and S4.4 (characterized by $\Diamond A\rightarrow(A\vee\Box\Diamond A)$), and, following Robert Stalnaker, that a particularly plausible candidate in this range is S4F (characterized by $\Box(\Box A\rightarrow B)\vee(\Diamond\Box B\rightarrow A)$). This is not a well-known logic and Humberstone reveals a surprising array of different axiomatic and semantic characterizations for it.

The book's penultimate chapter concerns the author's logic of coming about. This is a somewhat more abstract variant of the "sees to it that" logics that emanated from Pittsburgh a few decades ago. (See, for example, Belnap et al, 20001.) The chapter provides a simple semantics and axiomitizes the valid formulas, and makes some tentative but suggestive comments about distinguishing "ways" of coming to stand in a relation and how that idea might be related to issues about the intelligibility of cross-world and cross-time comparisons.

The final chapter is an appendix in which the author, borrowing from a number of sources, formulates natural deduction systems for a variety of modal systems. His aim is to get as close as possible to the ideal of a single "pure" introduction and elimination rule for each connective. The resulting systems are elegant and user-friendly, and it is somewhat odd that proofs within the body of the text make no use of them.

The book is not without flaws. One cannot help but think that a couple of the forgotten ideas that the author resuscitates and tidies in preparation for his final demolition might have been better left languishing in obscurity. The writing is generally lucid and entertaining, but the Faulknerian structure of some sentences may try the reader's patience. Though very few are likely to cause confusion, typographical errors abound. An errata sheet, including the errors spotted by the reviewer, is available online at ***. Readers looking for decisive answers to philosophical questions might be disappointed by the cautious, respect-every-position attitude adopted here. Nevertheless, overall this is an original and important book. It will richly reward the interested reader and it deserves a place on the bookshelf of every professional in the field.

REFERENCES

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