Review for J.S.L.
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Robert A. Bull and Krister Segerberg. 'Basic Modal Logic'. Handbook of Philosophical Logic. Volume II: Extensions of Classical Logic. Edited by D. Gabbay and F. Guenthner. D Reidel Publishing Company, Dordrecht, Boston and Lancaster, 1979, pp 1-88.

John P. Burgess. 'Basic Tense Logic'. Ibid, pp 89-134.

Richmond H. Thomason. 'Combinations of Tense and Modality'. Ibid, pp 135-166.

Johan van Benthem. 'Correspondence Theory'. Ibid, pp 167-248.

James W. Garson. 'Quantification in Modal Logic'. Ibid, pp 249-308.

Nino B. Cocchiarella. 'Philosophical Perspectives on Quantification in Modal Logic'. Ibid, pp 309-354.

These six papers constitute the most comprehensive and up-to-date survey of modal and tense logic now available. Reading them gives one a good sense of how much the field has developed in the last twenty years.

The papers can be divided into three pairs. Those by Bull/Segerberg and Van Benthem deal, for the most part, with propositional modal systems; those by Burgess and Thomason, with propositional tense systems, and those by Garson and Cocchiarella with predicate modal systems. I will discuss each of the three pairs, emphasizing the last.

'Basic Modal Logic' is really two papers. The first nine sections, written by K. Segerberg, contain an informative modern history of the subject, a catalogue of some of the more important modal systems, and brief discussions of consequence relations, semantic tableaux, and natural deduction systems. The discussion of natural deduction includes a sketch of a novel system for the logic K, in which the usual elimination and introduction rules are replaced by rules for elimination and introduction in modal contexts. For example, disjunction introduction permits the introduction of formulas of the form $\Box^n(A\vee B)$ given those of the form $\Box^n A$, and conditional introduction permits the derivation of $\Box^n(A\rightarrow B)$ from assumptions Γ when one has derived B from A and $\{G:\Box^n G\in\Gamma\}$. The suggestion provides an interesting characterization of K, though it would be somewhat unwieldy in application. The last fifteen sections, written by Robert Bull, comprise a rapid survey of much of the contemporary technical work in propositional modal logic. They include an especially detailed account of the relation between algebras and frames, a summary of techniques for obtaining irreflexive and asymmetric frames, sketches of completeness proofs for a number of systems including S4.3 and S4.1, and sketches of the proofs for all the following results.

1. Every normal extension of a transitive logic of finite depth (i.e., of a logic determined by the transitive frames (W,R) in which, for some n, any two elements are connected by an R-chain of at most n steps) has the finite model property.

2. Every normal extension of S4.3 has the finite model property.

3. There are exactly five pretabular extensions of S4, i.e., five normal systems containing S4, whose extensions are all determined by a single finite frame.

4. Where I_n is a formula expressing that a frame has width n (i.e., no more than n pairwise-incomparable worlds), there are uncountably many normal extensions of $S4I_2$. On the other hand, for every n all the normal extensions of $K4I_n$ are complete.

5. There are extensions L of KT and formulas A such that A is true in all finite frames for L, but not all frames for L. There are similar L and A such that A is true in all frames for L, but not all models for L.

6. There are extensions L of S4 and formulas A with the properties mentioned above.

7. There are extensions L of K and formulas A such that A is true in all normal neighborhood frames for L (i.e., all neighborhood frames for L in which the neighborhoods for every point constitute a filter), but not all models for L.

(The paper also includes proof sketches for several abstract frame-theoretic results that cannot be so easily summarized.) Sufficient detail is presented to give the flavor of the arguments and to make their truth plausible and references to the complete proofs are conscientiously supplied. The bibliography itself is of great value.

The paper on "correspondence theory" is a lucid and imaginative survey of a research program first delineated by its author. It has been known for a long time that certain modal formulas (like

$\Box p \rightarrow \Box\Box p$) correspond to certain first order formulas (like $\forall x \forall y \forall z(Rxy \wedge Ryz \rightarrow Rxz)$) in the sense that the modal frames verifying the former are exactly the first order models for the latter. Van Benthem is concerned with questions like the following. Which modal formulas correspond to first order formulas? Which first order formulas correspond to modal formulas? More generally, what classes of frames are defined by such formulas? How do the answers to these questions change when the truth conditions for $\Box$ are varied? How do they change when we restrict attention to transitive frames or other special classes? Are there similar interesting correspondences in tense logic? conditional logic? intuitionistic logic? modal predicate logic? Van Benthem sees much of the abstract work in modal logic as contributions to correspondence theory, and his paper contains summaries of some of this work as well as some suggestive forays in new directions. There is far too much here to summarize, but I will mention a few of the topics.

Some "results" in correspondence theory are very easy to obtain. There is an obvious correspondence between modal formulas and formulas of second order logic. For example $\Box p \rightarrow p$ corresponds to $\forall x \forall P(\forall y(Rxy \rightarrow Py) \rightarrow Px)$. In this case and many others, one can show that the second order formula is equivalent to a first order one. (Here, $\forall x Rxx$. The difficult direction is shown by substituting Rxu for Pu). In general, since the second order formula obtained is always Π^1_1, a result of classical model theory states that they will have first order equivalents if and only if they are preserved under ultraproducts. (With a little more work, van Benthem shows that this implies that modal formulas have first order equivalents if and only if they are preserved under frames that are elementarily equivalent first order models.)

Correspondence is distinct from completeness. K is complete for the class of irreflexive frames, but the K axiom does not correspond to irreflexivity. More interestingly, the McKinsey axiom $\Box\Diamond p \rightarrow \Diamond\Box p$ corresponds to no first order condition, although the logic it determines is known to be complete for some class of frames. Conversely, the frames for the axioms $\Box p \rightarrow p$, $\Box\Diamond p \rightarrow \Diamond\Box p$, and $\Diamond p \wedge \Box(\Box(p \rightarrow \Box p) \rightarrow p)$ are exactly the frames satisfying $\forall x \forall y(Rxy \rightarrow x=y)$, but that the logic with these axioms is not complete for any class of frames.

On tense logical frames, the formula $\mathbf{P}q \rightarrow \mathbf{GP}q$ corresponds to transitivity. On Lewis's counterfactual frames, the formula $(p \Box\!\rightarrow q) \vee (p \Box\!\rightarrow \neg q)$ corresponds to the linearity of the similarity relation. On Kripke frames for intuitionistic logic (i.e., general frames (W,R,P), where R is a partial order and P is the collection of all R-closed subsets of W), excluded middle corresponds to the formula $\forall x \forall y (Rxy \rightarrow x=y)$. These examples require only routine verification. A much more difficult argument is required to establish that Scott's Axiom, $((\neg\neg p \rightarrow p) \rightarrow (p \vee \neg p)) \rightarrow (\neg p \vee \neg\neg p)$, corresponds to no first order condition.

The paper by J. Burgess is a model contribution--succinct, comprehensive, and very readable. Burgess concentrates on completeness and decidability results in propositional tense logics. Axiomatizations are presented for time: partially ordered, totally ordered (with and without endpoints), dense, discrete, Dedekind-continuous, well-ordered, and lattice-structured. The completeness proofs presented are a clever variation of the usual construction. Instead of tinkering with the canonical model to make it conform to the desired properties, one associates maximal consistent sets with the elements of structures that already have the desired properties. One might expect to lose some advantages of uniformity, but in fact the differences in the completeness proofs for the various systems come down to differences in the proof of a single lemma. The discussion of decidability is not quite as detailed. The tense logic of lattice-structured time is shown to be decidable as an illustration of the method of filtration and the logic of rational time is shown to be decidable by reduction to Rabin's monadic second order theory of two successors. An argument that the decision problem for real time can be reduced to that for rational time is sketched, providing a very simple proof of decidability for the former.

The initial sections of Burgess's paper concern what might be labelled "traditional" topics in tense logic. The later selections contain some observations on some other topics. Gabbay's proof that in real time Kamp's **S** (<u>since</u>) and **U** (<u>until</u>) are sufficient to express all tenses is sketched and other recent work in this area is reported. There is a brief discussion of Kamp's **J** ("then") operator and the senses in which tense logics containing it can be reduced to those not containing it. And there are a few interesting observations about period-based systems, "metric" systems (permitting quantification over times and temporally indexed tense operators), systems for relativistic time, and systems for "thermodynamic time" (satisfying **GF**A → **FG**A, for all A that express atemporal, macroscopically observable facts).

In the second tense paper, Richmond Thomason argues persuasively for the importance of temporal considerations in formal treatments of necessity, obligation and conditionals. In the latter two areas very little technical work has been done. In the first area, some results have been obtained, but the methods are arduous. Consequently not much is actually proved in this paper.

The most detailed discussion concerns the notion of "historical necessity". This is the kind of necessity invoked in claims that the past is necessary and the future, contingent. It would seem to play a role in a number of important classical, medieval and modern discussions of determinism and practical reasoning. A reasonable way to understand this notion is to see time as a tree branching towards the future. Each branch is a possible future; one is the actual future. □A is true relative to a time t and a branch B if A is true at all branches B' through t. **F**A is true relative to t and B if A is true at some successor of t along B. These ideas are realized directly in what Thomason calls "tree-like" frames: structures (S,⊂,B) where ⊂ is a transitive, left-linear relation on S, and B is a branch through (S,⊂). But Thomason also considers, what he calls "T×W" frames: structures (W,T,<,≈,w), where < is a linear, irreflexive relation on the nonempty set T ("times"), and ≈ assigns an equivalence relation $\approx_t$ on the nonempty set W ("worlds") to every t∈T in such a way that $u \approx_t v$ implies $u \approx_s v$ for all s<t. (Intuitively $x \approx_t y$ iff x and y are identical through time t.) Thomason points out in a number of examples that one should not expect to obtain a satisfactory temporal theory of some phenomenon by simply amalgamating an atemporal theory of that phenomenon with a theory of time. It would seem that those who employ T×W frames are doing something like that with time and necessity, and Thomason makes it clear that he thinks the tree-like frames are the ones with real philosophical interest. In view of the close connections between the two kinds of frames, however, this may be unduly hard on the amalgamators. Every T×W frame (W,T,<,≈,w) is isomorphic to a tree-like frame. (Take S to be the all the equivalence classes $[w]_t$ for weW and t∈T and take $[u]_s \subset [v]_t$ if s<t and $[u]_s \stackrel{\perp}{=} [v]_t \neq \ddagger$.) Furthermore, if S is sufficiently homogeneous, tree-like frames (S,⊂,B) are equivalent to T×W frames. (For example suppose S is well-founded and serial (in the

sense that $\forall x\exists y(x\subset y)$). Then the branches of S are well-ordered sequences. (Let (S', $\subset$, B) be the subframe of (S,$\subset$,B) generated from B_1. Take W to be the branches in S, T and < to be natural numbers and the less than relation, and $u\approx_t v$ if $u_1=v_1,\ldots u_t=v_t$.) More generally, TxW frames seem to differ from tree-like frames only in requiring all worlds to be subject to a single clock. But temporal differences among different branches do not always make a difference to the logic. Even when they do make a difference, it is not obvious that the tree-frame picture is the correct one. There is nothing incoherent about the view that facts about the structure of time are necessary.

Efforts to provide explicit axioms for the logic determined by tree-like frames have not been successful, although Thomason reports a result of Gurevitch and Shelah that would imply its axiomatizability. Similarly, he reports an observation by Burgess establishing the axiomatizability of the logic determined by TxW frames, although its axioms have not been given. Somewhat better success is reported for a kind of frame that is intermediate between TxW frames and tree-like frames. In a "Kamp" frame worlds are required to share a common clock only while they coincide. Thomason lists axioms that, he says, Gabbay has recently proved sufficient for Kamp-validity. Even here there is work to be done, however. Gabbay's axiomatization includes a rather cumbersome "irreflexivity" schema. It is natural to ask whether such a schema is required. (Since most familiar modal systems are determined by classes of irreflexive frames, we know Gabbay's schema would add nothing to them.)

Modal predicate systems are notoriously more complicated technically and more problematic to interpret than propositional systems. In recent texts, if the subject is not omitted altogether, discussion is confined to a few systems, chosen more for their technical tractability than their philosophical interest. The papers by Garson and Cocchiarella thus fill an especially pressing need.

The language of modal predicate systems is obtained by simply adding the one place connective $\Box$ of necessity to ordinary predicate logic and perhaps a special unary predicate E of existence. In interpreting such a language, however, one faces a number of choices: Should possible worlds be mere "recombinations" of individuals and properties of this world or should they comprise genuinely different individuals? If the latter, should quantifiers range over all possible individuals or only the actual individuals, i.e., the individuals that exist in this world? Or should the object of quantifiers not be individuals at all, but rather concepts that pick out different individuals in different worlds. If the latter, should each model include the set of all possible such concepts or should we take as models "secondary" structures that specify certain collections of these (in the same way that Henkin's models for second order logic specify collections of sets of individuals over which property quantifiers may range)? Similar questions arise about the interpretation of terms and predicates. And a particular choice of interpretation for the simple constituents, may give rise to other questions about the interpretation of the expressions built from them. If a term t and predicate P denote in this world an individual and a set not part of this world, what should we say about the truth value of Pt?

Different answers to these questions give rise to a bewildering array of possible modal systems. Garson's paper contains a useful "roadmap" of a number of these systems that have been discussed in the literature. He concentrates on the question of whether the systems determined by particular interpretations are axiomatizable. For the systems that are axiomatizable (which seem to be those with individual quantification, those with secondary conceptual models and those whose with the modal principles of S5) Garson isolates four kinds of completeness arguments. Each of these is a variation on the Henkin argument, in which models for consistent sets are built from the formulas themselves, The difficulty in applying this kind of argument in the modal case is that the demand for "world-witnesses" for possibility sentences may conflict with the demand for "term-witnesses" for existential sentences. Once a world is constructed there will be no new terms left to witness an existential formula. Under some conditions (viz., when the stock of individuals is the same in every world) one can show that new terms are simply not needed--existential formulas are witnessed by old terms. This is strategy one. Strategy two is to build a whole sequence of worlds simultaneously, adding formulas that witness existential sentences to the incipient worlds in which they occur and formulas that witness possibility sentences to incipient worlds later in the sequence. On strategy three, term-witnesses are chosen from a new language. Truth-at-a world corresponds to membership only for formulas in the language of that world. Strategy four is to extend the applicability of strategy one by strengthening the quantifier rules. If universal generalization is replaced by the schema $\vdash A \rightarrow (\Box B_1 \rightarrow \ldots \rightarrow (\Box B_n \rightarrow Pt)\ldots)$ implies $\vdash A \rightarrow (\Box B_1 \rightarrow \ldots \rightarrow (\Box B_n \rightarrow \forall x Px)\ldots)$ when t is "new", then one can show in a wide variety of cases that existential formulas can be appropriately witnessed.

If individuals are identified with "rigid" concepts, i.e, concepts that pick out the same object in every world, then systems with objectual quantifiers, terms or predicates can be regarded as being obtained by adding special restrictions to a system with conceptual quantifiers, predicates and terms. Garson expresses the hope that a single completeness proof for the general system will yield axiomatizations for all the others. He suggests that strategy four comes close to providing such a "unifying" proof, but worries about the complexity of the quantifier rules required. It is not clear, however, that it is reasonable to expect a single strategy to succeed for all the diverse systems discussed. If the objectual systems are regarded as being obtained by adding restrictions to a conceptual system, it would presumably be restrictions to the unaxiomatizable primary conceptual semantics rather than to the secondary semantics. Garson does show that variations on strategy four have wide applicability and his suggestion that it would be worth trying to learn something about the conditions under which the complicated quantifier rules can be replaced by simpler ones seems to be a good one.

For systems that are not axiomatizable, Garson summarizes the incompleteness arguments. The idea is to use clever mappings to express the axioms of arithmetic in the system. In the case of predicate tense logics, one starts with a sentence expressing that each individual exists at exactly one time. The appropriate sentences about numbers can then be mirrored by tense logical formulas. Similarly in the case of modal logics with propositional quantifiers, one constructs a sentence expressing that each proposition is true at exactly one world, so that statements about numbers can be mirrored by modal statements. Garson shows how one can get the effect of propositional quantification in systems with quantifiers over individual concepts, so that the incompleteness carries over. All of these incompleteness results hold when the underlying modal system is S4.3 or weaker. Completeness results have been reported for systems whose logic is S5.

The fact that so many quantified modal systems have been considered in the literature probably reflects a divergence of views about what such a system is supposed to be about. Garson's discussions of the systems support this contention. A system is motivated by examples having to do with tense and then shown to be susceptible to "difficulties" that have to do with metaphysics or logic. The paper by Cocchiarella examines quantified modal systems from several more carefully delineated philosophical perspectives. Cocchiarella asserts that his aim is to examine claims that quantified modal logic may require commitment to essentialism, a "bloated" ontology of possibilia, or the rejection of contingent identity statements. Oddly, the semantics Cocchiarella considers first does not appear at all in Garson's roadmap. If we take <u>logical atomism</u> to be the view that the basic constituents of the world are primitive individuals, properties and relations, and that these are named by the constants and predicates of first order logic, then it is reasonable to identify possible worlds with <u>models</u> for predicate logic. The set of all models containing a particular domain of individuals and interpretations for a particular collection of predicates constitutes a "logical space" of such worlds. Thus on Cocchiarella's first semantics, $\Box A$ is true in a model for predicate logic, if A is true in all models for the language of A on the same domain. The resulting system is actually <u>anti</u>-essentialist. One can prove, for example, that $\exists x \Box Px \rightarrow \forall x \Box Px$. Thus no property holds necessarily of an object solely because of the nature of that object; if a property holds necessarily of one object it holds necessarily of all objects. Indeed if the domain of a model is infinite then, for nonmodal A, $\Box A$ is true if and only if A is logically valid. One can also prove that de re necessities are eliminable in the sense that every de re necessity is equivalent to a de dicto necessity. (Of course it might still turn out that de re necessity is a very <u>useful</u> concept because their de-dicto equivalents are complicated or because particularly broad or efficient proof procedures might require using de-re steps to prove de-dicto theorems).

On the standard contemporary interpretations for modalities, the "possibility space" is given by an arbitrary set W of primitive objects. This approach differs in at least two ways from Cocchiarella's "logical atomist" semantics. First, it allows the possibility of "duplicate" worlds, i.e., distinct worlds containing the same objects, properties and relations. Second, it allows the possibility that not all combinations of the basic objects, properties and relations are really "possible". As long as $\Box$ is taken to mean truth in all worlds, the first difference is insignificant. Adding or deleting duplicate possible worlds to a model always results in an equivalent model. To allow the second possibility, Cocchiarella, considers the secondary version of his semantics, according to which $\Box A$ is true at a predicate model with domain D if A is true at all the "possible" models with that domain. The resulting modal system is a familiar one--S5 modality with objectual quantification and a constant domain. It can be axiomatized by simply adding the S5 axioms to those for predicate logic. In this system, of course, formulas expressing anti-essentialism are not valid, nor can de re modality be eliminated. But no profound philosophical conclusions should be drawn from these facts. The primary semantics represents a logical atomist framework in which necessities are logical necessities, the secondary one represents an essentialist framework in which necessities may be metaphysical.

On both the frameworks discussed above, identity and non-identity statements with proper names and statements about the number of existing objects are either necessary or impossible. Thus these issues are independent of issues about essentialism. One can allow contingent identity and "census" statements by specifying for each world-model a subset of the domain of that model to represent the "existing" objects. A world still contains all individuals as constituents, but the range of quantifiers is restricted to the existing individuals. Once again, we can consider a primary version of the semantics, on which every possibility space must allow each subset of individuals to be the existing ones in some world, or a secondary semantics, on which an arbitrary collection of such subsets constitutes a possibility space. The first version is appropriate if one takes necessity to be a purely logical notion and existence non-logical and the second if one takes both necessity and existence to be non-logical.

Cocchiarella considers the two secondary semantics which do not validate the anti-essentialist theses to be more problematic than the primary semantics, which do. There is no reason to believe that <u>any</u> set of models on some domain can comprise a possibility space or that <u>any</u> set of individuals can be the set of existents. But whether the secondary semantics are coherent seems to turn more on our notion of validity than our notion of possibility. To say that $\Box A$ is <u>true</u> is perhaps to say that A is true in all possible worlds. But to say that $\Box A$ is <u>valid</u> is then to say that A is true in all the worlds of any structure. If the valid formulas are to be those true solely in virtue of form, and if one believes necessity and possibility to have real content, it would not seem to be appropriate to place restrictions on the sets of worlds or objects that can constitute a structure. (Of course a structure in this case might correspond to something that could never be realized.) When one takes the various names and predicates of a language to name basic, independent possibilia and relations, then the appropriate models are the primary ones. When one allows that predicates have independent forms, but contents that need not be independent, then the secondary semantics would seem to deliver the appropriate validities.

The "non-atomist" semantics, because they allow the set of existing objects to vary from world to world, can satisfy $c=d \wedge \neg\Box(c=d)$. But identity is still non-contingent in the sense that the sentence $\forall x\forall y(x=y \rightarrow \Box x=y)$ is valid. The latter formula, of course, fails when quantifiers range over individual concepts and Cocchiarella dutifully discusses this sort of semantics, which he labels "Platonic". Once again, however, the semantics has no real metaphysical consequences since the relation expressed by 'x=y' in this case is not really the identity of x and y, but the <u>coincidence</u> of x and y in a particular world. An example that does seem to support the view that identity can be contingent has been given by A. Gibbard. A certain lump of clay Lumpl is in fact identical to a certain statue Goliath. But if I were to squeeze Lumpl hard enough Goliath might cease to exist. So the identity between Lumpl and Goliath seems to be contingent.

Cocchiarella's analysis of Gibbard's example invokes what he calls a "socio-biologically based conceptualist view". The labelling here may be a little misleading since the formal semantics Cocchiarella advocates is purely objectual. The view is "conceptual" because proper names of natural language are not treated as directly referential individual constants, but rather as "proper name sortals", i.e., predicates that are true of one object d in every world in which d exists and true of nothing elsewhere. Thus the situation of the Gibbard example can be represented:

$\exists xL\ \exists yG\ (x=y) \wedge \Diamond \exists xL\ \forall yG\ x \neq y).$

where L and G are the proper name sortals corresponding to Lumpl and Goliath. The identity relation on objects remains non-contingent.

The book in which these papers appear is well-produced, but there are a few typographical errors including some that might cause confusion. Some corrections are listed below (negative line numbers are counted from the bottom of the page):

page 10 line -4. "algebra of sets" for "set of algebra"

page 118 line 16. "$x \in X$" for "$x \in K$"

page 119 line -12. "is" for "if"

page 121 line -1. "*x* is valid" for "is valid"

page 123 line 4. The first "**H**" should not be a subscript and the second "$\wedge$" should be "$\vee$".

page 127, line 1. The second symbol should be a "p".

page 129, line -15. The lower bound on n should be 0.

page 132, line 11. "Bull, R.A.: 1970" for "Bull, R.A.: 1978"

page 143, line 10. "$\Diamond\varphi$ " for "φ".

page 144, line 9. Omit the reference to chapter II.2

page 146, line 1. "¬**F**p" for "**F**¬p".

page 146, line -9. The last t should be a t'.

page 190, line 1. Definition 2.1.13 is missing.

page 200, line -3 "ψ" for "φ"

page 201, line -8. The reference is missing from bibliography.

page 231, line -4. "$\vee$" for "$\wedge$".

page 280, line -13. "{t':t=t' w}" for "{t:t=t' e w}"

page 302, line -14. "Q2-S5" for "Q2-Q5"