# Embedded Definite Descriptions: Russellian Analysis and Semantic Puzzles 

STEVEN T. KUHN<br>Department of Philosophy<br>Georgetown University<br>Washington, DC 20057 USA

(1) The man wearing the beret is French.

In this sentence one definite description is embedded within another. Employing a version of
Russell's iota notation for descriptions, ${ }^{1}$ the sentence can be represented

$$
\text { (1a) } F \imath x W^{2} x \imath y B y
$$

where " $F$ " and " $B$ " are unary predicate letters corresponding to ".. is French" and ".. is a beret" and " $W^{2}$ " is a binary predicate letter corresponding to " $\ldots$ is a man who wears $\qquad$ ". The common Russellian analysis of such sentences seems to produce questionable readings and miss the most natural one. I suggest an alternative. The project of providing a uniform procedure to

[^0]eliminate all embedded definite descriptions while preserving logical form, however, faces interesting difficulties.

Russell (1918 and 1919) does not mention the possibility of embedding descriptions in this way. Whitehead and Russell (1910) "amend" the simple iota notation in order to eliminate ambiguities in the scope of the description operators. For example, "The present king of France is not bald", which is rendered $\neg B \imath x K x$ in the original notation, is rendered either as $[\imath x K x] \neg B \imath x K x$, or as $\neg[\imath x K x] B \imath x K x$ in the amended. Whitehead and Russell do consider expressions in the amended notation like $\left[\lfloor x F x][\iota x G x] H^{2} \imath x F x ı x G x\right.$ in which one iota "quantifier" lies within the scope of another, but none that exhibit embedding within the iota "matrices" in the manner of (1a) above. Indeed, it is not clear whether and how (1) can be represented in the amended notation. Because they regard the notation of Principia as devices for expressing general truths about propositions, rather than as a formal object language that is itself worthy of serious study, Whitehead and Russell never bother to fully delineate the expressions that they consider well formed. There is some evidence, however, that they may have meant to exclude the kind of embedding we are considering. For example, they take some pains to prove a result described as establishing that "the truth value of a proposition containing two descriptions is unaffected by the question which has the larger scope." (See Whitehead and Russell (1910), p174. The proposition proved is $* 14 \cdot 113$ on page 176.) The proof, however, only covers the case of two descriptions that are "parallel," as they are in $[\imath x F x][\iota x G x] H^{2}{ }^{2} x F x ı x G x$. Indeed, as will be shown below, the result does not carry over in the most general way for embedded descriptions in the original notation.

Although Russell himself may have intentionally neglected notation like (1a), his
successors certainly do not. Modern treatments of the description operator (like those of Lambert and van Fraassen (1972), Burge (1974), Kalish, Montague and Mar (1980), Gamut (1991), and Lambert (1992) take " $x$ " to be an expression that applies generally to formulas to produce terms, so that expressions like (1a) (and those involving even deeper embeddings) are well formed. For the purposes of the discussion here let us accept this modern view while retaining Whitehead and Russell's "contextual definition" of the iota terms (Whitehead and Russell 1910, p173):

$$
\text { (CD) } \psi \iota x \phi \text { is to mean } \exists b[\forall x(\phi \leftrightarrow x=b) \& \psi b]
$$

Here $\psi t$ (where $t$ is a term or individual variable) indicates a formula with certain occurrences of t "marked" for possible replacement. We can take the "context" $\psi$ of the description occurrence to be a parameterized unary propositional function, that is a formula of predicate logic with identity and descriptions, one of whose free variables is singled out as argument, and the formula $\psi t$ to be the results of applying the functions $\psi$ to the term $t$.) ${ }^{2}$ We must also stipulate that $b$ does not occur in $\psi$ or $\phi$.

Each description occurrence in the formula on the right side of (CD) must lie within either $\phi$ or $\psi b$. For each occurrence in $\psi b$ there is a corresponding occurrence in $\psi \imath x \phi$. For each

[^1]occurrence in $\phi$ there are one or more corresponding occurrences in $\psi x x \phi$ (depending on how many of the occurrences of $x x \phi$ in $\psi$ are marked). Furthermore at least one occurrence of the description $\mathrm{x} \boldsymbol{x} \phi$ on the left side does not appear on the right. Thus any application of CD to a formula $\psi$ containing a description reduces the number of description occurrences. If CD is applied sufficiently many times to subformula occurrences within a formula $\theta$ that contain descriptions we obtain a formula containing no description occurrences whatsoever. Following the interpretation of Kaplan $(1972)^{3}$, let us call a description-free formula that results from such a series of applications of (CD) a logical translation of $\theta$. In general there may be many different sequences of applications of (CD) that culminate in logical translations. We can vary the order in which description occurrences are expanded, the subformulas of $\theta$ containing those description occurrences that are taken as $\psi \imath x \phi$ in a particular application of (CD), and the structure that $\psi u x \phi$ is presumed to have. (For example, if $\psi \iota x \phi$ is $\imath x \phi=\imath x \phi$, then it can be taken as either the application of the propositional function $(\imath x \phi=y)$ to $\imath x \phi$ or as the application of $(y=\imath x \phi)$ to $\imath x \phi$ or as the application of $(y=y)$ to $x \phi$. In each case application of (CD) would produce a different formula $\psi b$.) Let us say that $\theta$ is ambiguous if sequences of applications of (CD) produce more than one logical form and let us say that it is non-trivially ambiguous if they produce nonequivalent logical forms. The "king of France" example shows that some formulas are nontrivially ambiguous.

Most contemporary reconstructions of Russell's theory of descriptions either adopt Whitehead and Russell's amended notation or assign a particular reading to formulas in the

[^2]original notation. Even aside from the questions already mentioned about whether and how embedded descriptions can be represented in the amended notation, however, there are some good reasons to adopt the present framework. As Peter Geach pointed out fifty years ago (Geach 1950), the amended notation itself is at least trivially ambiguous, so that if univocal notation is considered important, its proper use really demands a proof that all logical translations of a given formula are equivalent.

Using the original Russellian notation with a fixed reading, on the other hand, makes it difficult to represent sentences where descriptions get the other readings. In most such accounts, for example, it is unclear how to represent the untrue ("wide scope") reading of "The king of France is not bald". More generally, to insist that we begin with an unambiguous formal language is to guarantee that we will be unable to represent the process by which grammatical form is converted into logical form. Since much of the philosophical interest of Russell's theory of descriptions derives from the idea that definite descriptions are paradigms of grammatical constructions that mislead us about logical form, it would seem important to ensure that we can represent that process.

Given this general Russellian understanding of descriptions, we can investigate the logical translations of (1a). There are a large number of ways that the descriptions can be eliminated which result in a somewhat smaller number of logical translations. As the careful reader may verify, however, all the logical translations are equivalent to the following:
(1b) $\exists b \exists m\left[\forall x(B x \leftrightarrow x=b) \& \forall y\left(W^{2} y b \leftrightarrow y=m\right) \& F m\right]$.
(1b) states that there is exactly one beret, there is exactly one man wearing that beret, and that man is French. There has been an extensive discussion of the kind of "uniqueness" required for colloquial uses of phrases of the form "the such and such". (See Roberts 1999 for an extensive survey and a defense of one alternative.) For example, the truth of "The light is on" seems not to require that there be only one light in the world, but perhaps that there be one light in some universe of most salient objects, or perhaps one light among some conventionally determined cache of available (real and imaginary) discourse referents. But no matter how we construe "uniqueness," it does not seem plausible to suppose that (1) says that there is a unique beret and that a Frenchman is wearing it. For imagine a room in which one beret is hanging prominently on a hat rack, a second is worn by a young girl and a third, somewhat less prominently, by a Frenchman. Suppose further that the prior conversation has concerned, say, the chances of life on other solar systems. Then (1) seems felicitous and true even though there is not a unique beret, or even one that is unique among the most salient or most discourse-relevant objects. A better representation of (1) is the following:

$$
\text { (1c) } \exists m \exists b\left[\forall x \forall y\left(W^{2} x y \& B y \leftrightarrow x=b \& y=m\right) \& F m\right]
$$

(1c) states that there is a unique pair of individuals whose second member is a beret being worn by its (adult human male) first member, and that the wearer is French. If, after hearing (1), someone asks "which man?" the proper response is "the one wearing the beret". If someone asks "which beret?" the proper response is "the one worn by the man." There may be other berets and there may be other men. What is unique is the pair consisting of a beret and a man wearing it. Furthermore, the appropriateness of this interpretation seems to depend on the descriptions being
embedded in the way we have been discussing. Sentences like "The beret is worn by the Frenchman" and "The Frenchman is wearing the beret" are not similarly felicitous and true in the imagined context.

The example suggests a modification of (CD) for translating pairwise definite descriptions.
(CD2) $\psi \imath x \phi_{1} \imath y \phi_{2}$ is to mean $\exists m \exists b\left[\forall x \forall y\left(\phi_{1} \& \phi_{2} \leftrightarrow x=b \& y=m\right) \& \psi m\right]^{4}$

The interpretation provided by (CD2) should be distinguished from two cousins. First consider the proposition expressed by $F \imath x \exists y\left(B y \& W^{2} x y\right)$, stating that the man who is wearing at least one beret is French. We may regard this as an embedding of an indefinite description. It is true when a man $m$ is wearing two berets $b_{1}$ and $b_{2}$ and no other men are wearing any berets. The pairwise description is false under these circumstances because $\left(m, b_{1}\right)$ and $\left(m, b_{2}\right)$ are distinct man-wearing-beret pairs. This observation might seem to undermine the analysis. But the intuition that one might truthfully say "The man wearing the beret is French" when the only capped individual is sporting two berets can be explained by pragmatic principles. Since the speaker did not employ the equally simple definite plural "The man wearing the berets is French", she must have believed that one of the two berets was more salient. This explanation gains plausibility when one considers cases in which differences in salience seem unlikely. Suppose we are looking at a display of the flags of the G7 countries. It does not seem that one can then felicitously and

[^3]truthfully say "The flag with the horizontal red stripe is American".

Now consider the proposition that there is exactly one man with the property of wearing exactly one beret and he is French. This might be expressed by the formula $F \imath x\left(W^{2} x \imath y W^{2} x y\right)$ "the man wearing the beret he is wearing is French" - where the first and second descriptions get widest and narrowest possible scopes. Like the pairwise definite description (and unlike the embedded indefinite description) this requires that there be a man wearing a unique beret. Unlike the pairwise definite description, however, it is not falsified by the presence of other, multiplybereted men. "The man wearing the beret is French" does not seem a felicitous and true description of a scene with one singly-bereted and several multiply-bereted Frenchmen.

The question of whether embedded definite descriptions are pairwise descriptions is independent of the question of whether they are used referentially or attributively and independent of the question of whether the referential use constitutes a second meaning for descriptions. (For a detailed and interesting discussion of these questions, see Neale 1990. If the speaker of (1) and her audience are sitting at a café observing the same parade of passing pedestrians it is likely that she intends to convey to the audience information about a particular individual, in particular, the information that he is French. She ensures that the audience ascribes Frenchness to the correct individual by identifying him as the first member of a noticeably unique "man-wearing-beret" pair. Indeed the pairwise description may be convenient because there are no more obvious ways to pick out that individual. On the other hand, suppose the speaker is talking by telephone, and has been told only that the listener is watching a group with a wide variety of headgear. In this case it would seem that there can be no particular individual whom
the speaker knows to be wearing a beret and to whom he intends to attribute the property of being French. Nevertheless the logical form of his utterance is more naturally construed as (1c) than
(1b). ${ }^{5}$

In the opening paragraph it was suggested that the treatment of descriptions proposed here is an alternative to the Russellian. There is some question whether sentences with the grammatical form of (1a) really can have a Russellian reading like that represented in (1b). The issue is difficult because the reading that I labeled "natural" is logically implied by the Russellian reading. ${ }^{6}$ Any conditions under which one can truly assert the Russellian reading of a sentence like (1) will be conditions under which one can also assert the natural (i.e. pairwise definite) reading. Furthermore, many examples in which the stronger reading might seem to be implied by the sentence can be explained as examples where the stronger reading can be inferred from the context. There is an analogy here to the familiar question of whether the word "or" has an exclusive reading. It is not sufficient for the defender of exclusive "or"'s to point out that we commonly assert truths like "Today is Wednesday or Thursday" on Wednesdays and Thursdays, because that sentence would be true on either interpretation of "or". It is also difficult for him to insist that the assertion, rather than common knowledge, implies that today is not both

[^4]Wednesday and Thursday. Similarly, it is not sufficient for the defender of Russellian embedded descriptions to point out that we truthfully assert "Nine is the square of the successor of two", and when we do our audience understands that two has a unique successor and that nine is its unique square.

Additional evidence to compare the treatment of embedded descriptions under CD and CD2 comes from cases in which the variable of the embedding description binds an occurrence in the embedded one. Consider

> (2) The man wearing the beret he made is French,
which we might represent

$$
\text { (2a) } F \mathfrak{l} x W^{2} x u y B^{2} y x
$$

( $F$ and $W^{2}$ are as before and " $B^{2}$ " is a binary predicate corresponding to " $\ldots$ is a beret made by _".)

Applying CD2, we get

$$
\text { (2b) } \exists b \exists m\left[\forall x \forall y\left(W^{2} x y \& B^{2} y x \leftrightarrow x=m \& y=b\right) \& F m\right]
$$

which states that there is a unique pair ( $\mathrm{b}, \mathrm{m}$ ) such that m is a man who made and is wearing beret $b$ and that the second member of this unique pair is French. (It does not require that the man have made no other berets, nor that he be wearing no additional berets made by another.) This seems a reasonable logical translation of (2).

We can contrast (2b) with some of the logical translations obtained by (CD). Shown
below are three sequences of steps by which CD eliminates descriptions in (2a).

$$
\begin{array}{ll}
\text { (2c) } & \exists m\left[\forall x\left(W^{2} x y y B^{2} y x\right) \leftrightarrow x=m \& F m\right] \\
& \exists m\left[\forall x\left(\exists b\left[\forall y\left(B^{2} y x \mapsto y=b\right) \& W^{2} x b\right] \mapsto x=m\right) \& F m\right] \\
&  \tag{2d}\\
\text { (2d) } \quad \exists m\left[\forall x\left(W^{2} x y y B^{2} y x\right) \leftrightarrow x=m \& F m\right] \\
& \exists b\left[\forall y\left(B^{2} y x \leftrightarrow y=b\right) \& \exists m\left[\forall x\left(W^{2} x b \hookleftarrow x=m \& F m\right]\right]\right. \\
& \\
\text { (2e) } \quad \exists m\left[\forall x\left(W^{2} x l y B^{2} y x \mapsto x=m\right) \& F m\right] \\
& \exists m\left[\forall x\left(\exists b\left[\forall y\left(B^{2} y x \mapsto y=b\right) \&\left(W^{2} x b \leftrightarrow x=m\right]\right) \& F m\right]\right.
\end{array}
$$

In each of these translation sequences, the first step eliminates the outer description operator. The second step eliminates the inner description operator, taking different portions of the formula as its context. In (2c) the inner description operator is assigned the smallest possible context. The resulting formula states that there is a unique man with the property of making exactly one beret and wearing it, and that man is French. In (2d) the inner description operator is assigned maximal context. The resulting formula is a propositional function in $x$, stating that $x$ made a unique beret, and that exactly one man wore it. It seems clear that the last step in this translation sequence constitutes a pathological application of CD that the Russellian did not intend to permit. When descriptions contain variables that are bound by external quantifiers or external description operators, the Russellian may wish to restrict the application of (CD). He may, perhaps, wish to disallow any applications that produce formulas with new free variable occurrences. In (2e) the inner description operator is assigned an intermediate context, one that includes part, but not all,
of the formula obtained by expanding the outer description. This application of CD does not violate the restriction just contemplated. It produces a formula stating that everybody makes exactly one beret (call it "his" beret), exactly one man wears his beret, and that man is French. This translation is logically independent of the pairwise one. For suppose that Jean Claude, the sole beret maker, is wearing one of the two berets he has made and all others are hatless. Then the pairwise sentence is true, but this one is false. And suppose that it is the norm among Frenchmen to make and wear many berets, but poor Jacques has managed to make only the one he is wearing. Then the pairwise sentence is false, but this Russellian one is true. The reading (2e) might perhaps be a plausible reading of (2), though probably not one contemplated by defenders of Russellian analyses of descriptions. ${ }^{7}$ Whether or not (2c) and (2e) are plausible readings of (2), they provide a counterexample to the general claim that the truth values of formulas with two description operators is independent of the translation procedure by which the descriptions are eliminated.

CD2 provides a logical translation for pairwise definite descriptions, but it does not provide what Russell wanted - uniform procedure by which all occurrences of descriptions can be eliminated. In the remainder of this paper I would like to examine some of the issues that arise in trying to generalize CD 2 to get such a procedure.

It is relatively easy to imagine a generalization in depth. Consider:

[^5](3) The man wearing the beret with the button is French.

This sentence does not to rule out multiple men, multiple berets or multiple buttons. Nor does it rule out multiple men wearing berets or multiple berets with buttons. What it does seem to rule out is multiple triples $(m, b, n)$ such that $m$ is a man wearing beret $b$ with button $n$. This idea can be made precise by a general translation schema for multiply-definite descriptions:
(Cdn) $\psi t x_{1} \phi_{1 \ldots} x_{\mathrm{n}} \phi_{\mathrm{n}}$ is to mean $\exists b_{1} \ldots \exists b_{\mathrm{n}}\left[\forall x_{1} \ldots \forall x_{\mathrm{n}}\left(\phi_{1} \& \ldots \& \phi_{\mathrm{n}} \leftrightarrow x_{1}=b_{1} \& \ldots \&\right.\right.$

$$
\left.\left.x_{\mathrm{n}}=b_{\mathrm{n}}\right) \& \psi b_{1}\right]
$$

The general schema could be replaced by two specific rules by introducing an intermediate notation for pairwise definite descriptions and taking advantage of the reduction of n-tuples to pairs. Let us understand b and $\overrightarrow{\mathrm{y}}$ to be the n -tuples $\left(\mathrm{b}_{1}, \ldots, \mathrm{~b}_{\mathrm{n}}\right)$ and $\left(y_{1}, \ldots, y_{\mathrm{n}}\right)$, identify the pair $(\mathrm{b}, \mathrm{b})$ with the $n+1$ tuple $\left(b, b_{1}, \ldots, b_{n}\right)$, and identify one-tuples of variables with the variables themselves.
$\left(\mathrm{CDM}_{\mathrm{i}}\right) \psi \iota x \phi_{1}\left(x, \mathrm{y} \overrightarrow{\mathrm{y}} \phi_{2}\right)$ is to mean $\psi \imath(x, \stackrel{\rightharpoonup}{\mathrm{y}})\left(\phi_{1}\left(x, y_{1}\right) \& \phi_{2}\right)$
$\left(\mathrm{CDM}_{\mathrm{ii}}\right) \psi\left(\overrightarrow{\mathrm{y}} \phi\right.$ is to mean $\exists \mathrm{b}\left[\forall \overrightarrow{\mathrm{y}}(\phi \mapsto \mathrm{b}=\overrightarrow{\mathrm{y}}) \& \psi b_{1}\right]$

Here $\phi\left(x_{1}, \ldots, x_{\mathrm{n}}\right)$ indicates a formula with occurrences of $x_{1}, \ldots x_{\mathrm{n}}$ marked for simultaneous replacement. For example, translating (3) into Russellian notation, we get

$$
\text { (3a) } F \imath x W^{2} x \imath y B^{2} y ı z N z
$$

where " $F$ " and " $W^{2}$ " are as before, " $B^{2}$ " is a binary predicate corresponding to " $\ldots$ is a beret with
$\qquad$ on it" and " $N$ " is a unary predicate corresponding to "...is a button". Applying the two
clauses of CDM to (3a) we get

$$
\begin{align*}
& F \mathrm{l} x W^{2} x \mathrm{l}(y, z)\left(B^{2} y z \& N z\right)  \tag{3b}\\
& F \mathrm{l}(x, y, z)\left(W^{2} x y \& B^{2} y z \& N z\right) \\
& \exists(b, c, d)\left[\forall(x, y, z)\left(W^{2} x y \& B^{2} y z \& N z \leftrightarrow(b, c, d)=(x, y, z)\right) \& F b\right],
\end{align*}
$$

which gives the reading desired. More informally, we take the second and third descriptions to assert the existence of a unique beret, button pair and then take the first to assert the existence of a unique man, beret-with-button pair.

It is somewhat more difficult to see how CD2 should be generalized to admit definite descriptions of arbitrary width. First, consider
(4) The man wearing the beret and carrying the newspaper is French, which we can represent
(4a) $F \mathfrak{l} x\left(W^{2} x \downarrow y B y \& C^{2} x \downarrow y N y\right)$.
Surprisingly perhaps, this seems to fit the same sort of logical translation rule as the case in which the third description is embedded within the second:

$$
\text { (4b) } \exists m \exists b \exists n\left[\forall x \forall y \forall z\left(W^{2} m b \& C^{2} m n \leftrightarrow x=m \& y=b \& z=n\right) \& F m\right) \text {. }
$$

There may be many salient men, berets, newspapers, men wearing berets, and men carrying newspapers, but there is only one man-beret-newspaper triple whose first member is both wearing its second and carrying its third. Because there are so many different forms that "parallel" embedded descriptions can take, it would be rash to generalize on the basis of this one example. One interesting case is where the clauses containing the embedded descriptions are joined by a
disjunction rather than a conjunction. The translation rule above renders all such sentences inconsistent. For suppose, for example, that there was a unique triple ( $\mathrm{m}, \mathrm{b}, \mathrm{n}$ ) such that m is either wearing $b$ or carrying $n$. If $m$ is wearing $b$ then that triple cannot be unique after all, because any triple ( $m, b, x$ ) will satisfy the required condition. Similarly, if $m$ is carrying $n$, any triple ( $m, x, n$ ) will satisfy the required condition. So there can be no such unique triple. "The man wearing the beret or carrying the newspaper is French" does seem odd in most contexts ${ }^{8}$, but it does not seem inconsistent. Instead, it seems to have the truth conditions that we might associate with a pairwise description. $\exists m \exists z\left[\forall x \forall y\left(\left(\left(W^{2} x y \& B y\right) \vee\left(C^{2} x y \& N y\right)\right) \leftrightarrow m=x \& z=y\right) \& F m\right)$. This asserts the existence of a unique pair whose first member is a man and whose second is either a beret the man is wearing or a newspaper that he is carrying. Notice that this is stronger than the disjunction "Either the man wearing the beret is French or the man carrying the newspaper is French", which is compatible with there being one man wearing a beret while others are carrying newspapers.

It is puzzling that two descriptions embedded within a third should express a pairwise definite description when connected by "or" a triply definite description when connected by "and". Even more puzzling is the semantic behavior of embedded descriptions modified by negations. Consider what we might colloquially express:
(5) The man without the beret is German.
which can represented
(5a) $G u x \neg W^{2} x u y B y$.

[^6]It is difficult to see how (5) can be construed as a multiply definite description. The question "What beret?" asked after (5) has no obviously correct reply. To eliminate the inner description in favor of quantification over phantom unworn berets is to do exactly what Russell wanted to avoid. Moreover, the unwanted apparent reference cannot be eliminated by any simple scope adjustments of the sort Russell employed to explain "The King of France is not bald". (5) does, of course, have a proper Russellian reading: there is exactly one (salient) beret and one (salient) man not wearing it and he is German. The more natural reading this time, however, is that each man is wearing exactly one beret except for one man who is wearing none, and that man is German. Again, the embedded definite description does seem to signal a kind of uniqueness. When some of the other men are wearing multiple berets it seems more appropriate to use the indefinite article. In a common forest scene, "The tree without a leaf is dead" is felicitous while "The tree without the leaf is dead" is not.

The examples considered in this paper suggest that sentences containing embedded descriptions can, as Russell would have wanted, be plausibly represented by logical formulas that contain no singular terms corresponding to the descriptions. In many cases this representation takes the sentences to exhibit what we have called "multiply definite" description, which generalize Russell's analysis in a straightforward way. Other examples, however, suggest that a more general procedure for obtaining logical translations of sentences with embedded descriptions faces greater difficulties. ${ }^{9}$

[^7]
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[^0]:    ${ }^{1}$ For readability, Russell's notation is replaced by a more modern variation throughout the paper. Iotas replace inverted iotas. Capital roman letters for specific predicates (with numerical superscripts indicating degree when they are not unary) often replace Russell's lower case Greek letters for propositional functions. Dot notation is eliminated in favor of parentheses and brackets. Parentheses around description operators and those around their atomic arguments are dropped, so that $B(\imath x)(F x)$ becomes $B \imath x F x$. Subscripts on biconditional signs are replaced by the appropriate quantifiers. On the other hand, the modern convention of allowing only letters from the end of the alphabet to serve as individual variables is flouted. Further changes in Russell's notation will be discussed subsequently.

[^1]:    ${ }^{2}$ The function must be unary so that we know which argument place $t$ is to fill. The parameters are necessary to permit quantification into descriptions. This will be needed if the notation is to be adequate to represent sentences like "Every Englishman honors his mother" (as the proponents of the modern view certainly intend) and "The square of an even number is even" (as Whitehead and Russell themselves apparently intend). The $\phi$ here replaces Whitehead and Russell's $\phi x$. There is no need to mark any occurrences of " $x$ " since they all remain unchanged. Because the occurrences of " $x$ " that are bound by the description operator in $\psi u x \phi$ correspond exactly to those that bound by the universal quantifier in $\exists b[\forall x(\phi \leftrightarrow x=b) \& \psi b]$, no rewriting of bound variables is necessary.

[^2]:    ${ }^{3}$ Alternative interpretations which Kaplan finds less satisfactory are that (CD) constitutes a definition of definite descriptions and that definite descriptions are abbreviations for which (CD) provides rules of elimination.

[^3]:    ${ }^{4}$ Here it is required that $y$ have no free occurrences in $\phi_{1}$. In order that CD2 might provide a full elimination procedure we may understand that if $y$ does occur free in $\phi_{1}$, it permits the bound occurrences of $y$ in $l y \Phi_{2}$ to be relettered as a new variable.

[^4]:    ${ }^{5}$ Similarly, although this paper takes a Russellian approach, it should be clear that the question of whether there are pairwise definite descriptions is independent of the abundantly scrutinized issue of whether the descriptions imply, or merely presuppose, the uniqueness conditions. See Neale (1990) for discussion of and references to this issue.
    ${ }^{6}$ Hornstein and Pietroski (1999) argue generally that determinations of ambiguity are difficult theoretical judgements not to be made by easy appeals to linguistic intuition and that the determinations are particularly difficult when one putative reading logically implies the other. They maintain that many of the widely believed judgements of this kind (including the ambiguity of "Every boy loves a girl") may be mistaken.

[^5]:    ${ }^{7}$ A similar phenomenon was noted in Heim (1991). Heim sketches an account of how we modify contexts when we find that they are inconsistent with the application of general rules for computing presuppositions of newly introduced sentences. Her general account allows a reading of "A fat man was pushing his bicycle" on which it presupposes that every man has a bicycle. She provides reasons, however, why other "accommodations" are preferred.

[^6]:    ${ }^{8}$ In particular the sentence seems felicitous when it is understood from context that there is a man who is wearing a beret or carrying a newspaper, but it is not known which. Such circumstances are probably rare, but there are more widely acceptable examples with similar structure. "The man sitting in the first or the second chair is French" seems to assert the existence of a unique pair $(\mathrm{m}, \mathrm{c})$ such that m is sitting in c and c is the first or second chair.

[^7]:    ${ }^{9}$ Anthony Ungar, Matt Burstein and members of the Georgetown/Maryland Logic and Language Group provided helpful comments on an earlier version of this paper.

