A New Introduction to Modal Logic

By G.E. Hughes and M.J. Cresswell

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To a generation of philosophers, modal logic was a field with ties to some of the most active areas of research and Hughes and Cresswell was the authoritative text and reference to that field. 'Hughes and Cresswell,' of course, referred to the authors' An Introduction to Modal Logic, published by Methuen in 1968 and reprinted as a University Paperback in 1972. Supposing no prior logic, An Introduction presented, in remarkably readable prose, axiomatic and semantic treatments of the propositional modal systems T, S4 and S5 (including completeness and decidability results), similar treatments of a few basic predicate systems, and an extensive survey of other results. Although this work continues to be widely used and cited, it has become somewhat dated. There are now more streamlined methods of proving completeness and decidability than the normal forms and semantic diagrams it employed. There has been a realization that the modal 'frame' is a more fundamental semantic notion than 'model' and a concomitant investigation of the phenomenon of modal 'incompleteness' (i.e., of systems not characterised by classes of frames). And there has been an explosion of further investigations and applications beyond those contained in the original survey. Several newer texts have appeared, but none combines the lucidity and the encyclopedic perspective of Hughes and Cresswell. Brian Chellas's excellent Modal Logic: An Introduction, for example, treats only the propositional systems. Hughes and Cresswell themselves tried to fill the void with A Companion to Modal Logic (Methuen & Co., 1984). This was a self-contained work in the same style, but at a more advanced level, than the previous. To use both an 'introduction' and 'companion' as texts is somewhat awkward, however, and the Companion itself has become dated. Thus, the New Introduction fills a great need. The relevant parts of An Introduction and A Companion have been rewritten and integrated with a wealth of new material into a worthy successor to the original Hughes and Cresswell.

Like its predecessor, A New Introduction supposes no prior logic. It comprises two parts on propositional systems, a third on quantificational, a couple of dizzying tables cataloging the modal systems discussed and a selection of solutions and partial solutions to exercises. Part One begins with a discussion of classical logic and modal systems K, T and D. (The earlier volume had begun only with T, presumably because that is the weakest system in which the modal operator can plausibly be regarded as a kind of necessity.) S4 and S5 are characterised in a natural way by considering which reduction principles might reasonably be added to T. As a hint of the rich variety of systems that await in subsequent chapters, the system B is characterised and the inclusion relations among all these basic systems are established. We get a more detailed outline of the modal landscape by learning that the systems Triv (obtained by adding $p \leftrightarrow \Box p$ as an axiom to K) and Ver (obtained by adding $\Box p$ as an axiom), and (less trivially) only these, are maximal. Much of the material from An Introduction on conjunctive normal forms in S5 and the method of diagrams is retained, but it is now supplemented by a chapter on canonical models that provides quick completeness proofs for all the basic systems. There is a new emphasis in text and exercises on admissibility of rules, which is now an active area of research.

Part Two, in just over a hundred pages, provides a clear and detailed account of "mainstream" modal logic, developed over the last thirty years, and a cursory look at some of its more interesting tributaries. In the former category, we get completeness proofs for the key extensions of S4. We get proofs of the finite model property (via sets maximal relative to the set of subformulas of the formula to be satisfied) and its absence (via an adaptation of Makinson's proof.) We get a proof that the "Gödel" logic ($K+\Box(\Box p \rightarrow p) \rightarrow \Box p$) is complete but not canonical. We get an example of a fairly simple incomplete logic. (A Companion used a clever but gerrymandered example of van Benthem. Here we see Boolos's more natural $K+\Box(\Box p \leftrightarrow p) \rightarrow \Box p$, which turns out to be validated by exactly the frames that characterise the aforementioned Gödel logic.) We get a discussion of "correspondence" theory and examples of complete systems that can and cannot be characterised by first order conditions on frames. In the latter category, we get an abridged version of the earlier survey of strict implication, some material on natural deduction, multi-modal systems, neighborhood semantics, program logics, intermediate logics and several other topics.

An Introduction and A Companion both restrict discussion of quantificational modal logic to systems in which quantifiers range over, and predicates apply to, a fixed domain of objects. Such systems validate both the Barcan formula $(\forall x\Box A\rightarrow\Box\forall xA)$ and its converse. Part III begins with a treatment of these simple cases, but goes on to explore (in varying detail) several alternatives. In the simple cases axiomatisations can sometimes be obtained by simply adding the Barcan formula and classical quantificational axioms to the appropriate modal base. Sometimes, however, adding Barcan and quantification to a modal logic produces incomplete systems. A New Introduction nicely demonstrates this phenomenon in the case of $S4+(\Box\Diamond A\rightarrow\Diamond\Box A)$ and points to other examples in the literature. It is not difficult to extend the simple conception of quantificational logic to systems in which domains may expand in accessible worlds (so that objects that do not actually exist might do so and Barcan fails). A New Introduction chooses to allow predicates to express relations among mere possibilia, while maintaining actualist quantifiers. The case in which domains may contract (so that actual objects might not exist and Barcan's converse fails) is somewhat more problematic. Here several alternatives are examined, but the preferred choice is again to keep actualist quantification and possibilist predication and to add to the language a predicate of existence. A chapter is devoted to the addition of identity and description operators to such systems and another to replacing objectual quantifiers and predicates by those applying to individual concepts. Other topics from the recent literature are briefly surveyed in a final chapter.

Modal logic is too extensive to be surveyed as exhaustively as it was in An Introduction, and proofs are sometimes too difficult to be as easily explained. Given these limits, A New Introduction provides an exceptionally clear and thorough picture of the field. Proofs are quite informal, and occasionally one feels the need for more detail. (This occurs mostly when the authors mean to be merely sketching ideas that they find peripheral to their main line of exposition.) There appear to be relatively few misprints and other errors. A short list of these, few likely to cause confusion, is available from the reviewer on request.

Perhaps the most extraordinary aspect of the book is the way the authors have digested, organised and explained so many bits of work from around the world and scrupulously kept track (in text and notes) of who did what. This meticulous compilation, combined with synoptic perspective and lucidity of exposition, will ensure that Hughes and Cresswell retains its status as the bible of modal logic.

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