MODAL LOGIC: AN INTRODUCTION. By Brian F. Chellas. Cambridge University Press; New York. 1980. Pp xii, 295. $42.50 (hardbound), $14.95 (paper).

Of the half dozen or so modal logic books now on the market, this is clearly the textbook of choice for an introductory course. Its use of Henkin style arguments to obtain streamlined completeness proofs for many modal systems and its use of filtrations to obtain similarly streamlined proofs of the finite model property sets it apart from the previous standard, Hughes and Cresswell and other books of that vintage. The clear and careful style of writing and the obvious concern for pedagogy set it apart from other new texts.

The organization of the book reflects the author's concern for pedagogy. The more specific topics are introduced before the more general ones. One learns about S5-necessity (as truth in all possible worlds) in Part I. "Standard" models (i.e., models with a world-world accessibility relation) are introduced in Part II. Finally, "minimal" (i.e., neighborhood) models are introduced in Part III. This insures that students will not be put off by excessive abstraction early in the game. Within each part one learns first about formulae valid in various classes of models, then about formulae derivable in various axiomatic systems and then (after being quite ready to believe it) one proves that the systems are complete with respect to the classes of models. There is a great deal of intentional overlap: a topic may be covered two or three times at successively deeper levels. An exercise on page 12, for example, establishes that the schema $\Box A \rightarrow A$ is valid in reflexive standard models and another exercise on page 40, that this schema can fail in non-reflexive standard models. These facts are explicitly confirmed in the text on pages 78-81. This "layered" style makes the book a little less handy as a reference. For example, there are twenty different places at which one can read about the system $\mathbf{K}$ But that disadvantage is minimized by the inclusion of a detailed table of contents, a general index, an index of symbols and an index of schemas rules and systems. Exercises are interspersed frequently throughout the book. There are several hundred in all. These frequently preview material that is introduced in subsequent sections of the book, but they also serve to introduce important supplementary material. For example, the p-morphism theorem and the connection between propositional S5 and monadic predicate logic are each the subject of an exercise, and algebraic models are investigated in a series of exercises that is sustained through several chapters. For the most part Professor Chellas is content to develop his subject, avoiding both history and philosophical applications. He does give the students a taste of the kinds of applications that are possible, however, by including a chapter on moral obligation at the end of Part II and a chapter on conditionals and conditional obligation at the end of Part III. This material is taken from the author's own work on the subjects.

In addition to being well organized, the book has been written very carefully, with attention to detail that should make a student's life easier. By taking the logical symbols to be names of linguistic entities and operations on such entities rather than the entities and operations themselves, Chellas completely avoids using quotation marks. The use of Greek letters has been minimized. Within each chapter results and definitions are numbered consecutively. Cross references are given whenever helpful. Results about the relative strength of systems, the strengths of modalities within systems and the links between systems and classes of models are summarized nicely in diagrams and charts. There are very few misprints or mistakes of any consequence. An errata sheet compiled by the author had thirty-four entries at the time this review was written, but these were mostly items like "change ´Also´ to ´Moreover´". The few errors that could cause confusion will be listed in this review.

As one would expect in an introductory textbook, the bulk of the material in the book is well known. There are, however, a number of innovations that deserve mention. Foremost among these are improvements in the completeness and finite model property proofs. On the standard approach one proves completeness for simple systems by constructing a canonical model that satisfies an arbitrary consistent set of sentences and (if necessary) tinkering with the canonical model until it satisfies the axioms for the logic in question. Finally one shows that the tinkering has not affected the relation between the model and the consistent set. Similarly, to prove that a system has the finite model property, one takes the gamma-filtration of a model for some consistent set gamma, tinkers with it until it meets the appropriate conditions and then shows that the tinkered model still satisfies gamma. Chellas extracts the conditions that the tinkered model must meet to avoid deviating detrimentally from its source and uses these to define a more general notion of 'canonical model' and 'filtration'. More specifically a canonical standard model, in Chellas's sense, satisfies the condition: $\Box$A is in alpha if and only if for every beta such that alpha R beta, A is in beta. A canonical minimal model for sigma satisfies the condition: $\Box$A is in alpha if and only if the proof set of A is a neighborhood of alpha. A filtration of a standard model satisfies the condition: [alpha] R* [beta] implies that if $\Box$A is true at alpha in the original model then A is true at beta, and if A is true at beta then $\Diamond$A is true at alpha. A filtration of a minimal model through gamma satisfies a condition which says that if $\Box$A is in gamma then the truth set of A in the original model is a neighborhood of alpha just in case its equivalence class is a neighborhood of alpha in the filtrated model. One can then show that any canonical model satisfies the formulae which belong to its worlds and any filtration of a model through gamma agrees with that model on the formulas of gamma. This means that to prove that a system is complete or that it has the finite model property it is sufficient to find a canonical model or a filtration model of the appropriate kind.

In addition to this tidying up of the completeness and finite model property proofs, a number of other innovations are included. To show that familiar modal logics are determined by classes of irreflexive, assymmetric and intransitive models (in addition to the classes of models they are usually associated with) Chellas uses a construction of Sahlquist. This provides a much shorter proof than the more pedestrian method of making reflexive transitive and symmetric models irreflexive, intransitive and assymmetric by replacing inappropriately related worlds by "copies" of themselves. Also included is the comprehensive completeness theorem of E.J. Lemmon for logics obtained by adding to K axioms of the form: $\Diamond^k \Box^l A \to \Box^m \Diamond^n A$.

This proves completeness for nearly all the well known systems in one fell swoop. By using a clever notation for properties of relations, Chellas makes the result seem even slicker.

In addition, there are comprehensive accounts of duality, of the number and strength of the modalities in various systems and of alternative axiomatizations for many familiar systems. Most of this material is probably known, but I do not know of any place where it is compiled as carefully as it is here. The exercises contain some interesting hints about alternative modellings. In one, the accessibility relation is replaced by a function f from worlds to sets of worlds. $\Box$ A is true at a world alpha if the truth set of A includes f(alpha). In another (which was also mentioned by Richard Montague in 'Pragmatics') the accessibility relation is replaced by a set of such relations. $\Box$A is true at alpha if there is some relation R in the set such that A is true at all R-related worlds.

The most obviously original material is contained in the discussions of obligation and conditionality alluded to earlier. Chellas's approach is to try to find an account so general that it is compatible with anyone's theory of obligation or any kind of conditionality. This account will then reflect the naked logic of the notions in question. In the case of obligation the 'standard' logic of obligation is axiomatized by adding the axiom $\Box T$ to the system K (I am using "$\Box$" to mean "it is obligatory that".) Chellas considers various strengthenings of the standard logic. Deontic S5 he rejects on grounds that it implies that "what is permissible ought to be permissible". He is more attracted by the system K + $\Box T$ + $\Box A \rightarrow A$. The possibility of combining obligation operators with tense operators is discussed, although no complete axiomatizations are given. In the end Chellas rejects all deontic systems based on the standard logic because they contain $-(\Box A \ \& \ \Box - A)$. This principle is incompatible with moral theories that admit conflicting obligations. In place of the standard deontic logic Chellas suggests we adopt a "minimal" deontic logic axiomatized by the rule $A \rightarrow B / \Box A \rightarrow \Box B$ and the axiom $- \Box F$. In the case of conditionality, Chellas suggests that a conditional A => B can be viewed as saying that B is in some sense necessary relative to A. Accordingly we can say that A => B is true at a world alpha if B is true at all worlds A-related to alpha. This analysis gives rise to the "basic" conditional system CK, axiomatized by the two rules:

$$A \rightarrow A' / (A \Rightarrow B) \rightarrow (A' \Rightarrow B)$$

$$(B1 \ \& \ldots \& \ Bn) \rightarrow B / ((A \Rightarrow B1) \ \& \ldots \& \ (A \Rightarrow Bn)) \rightarrow (A \Rightarrow B).$$

Since there is no single kind of conditionality to which this system is supposed to correspond it is difficult to assess the merit of this account of conditionality. In its favor is the fact that the material conditional, the strict conditionals and the counterfactual conditionals of Lewis and Stalnaker all satisfy the CK .

The best evidence for the utility of CK is provided by Chellas himself who shows that a reasonable conditional obligation operator O(A/B) can be defined as A=>OB where O is the minimal obligation operator and =>, the basic conditional. This approach would seem to make more sense than the usual one whereby conditional obligation is taken as a primitive. It would not be possible if stronger notions of obligation or conditionality were used.

If there is a weakness in the book, it is probably its limited scope. The most egregious omission is quantifiers. We are told in the preface that the book was originally to be a joint project with Chellas writing the propositional chapters and Lee Bowie writing the chapters about quantifiers. Presumably the two halves will now be published separately. This division seems logical. Propositional modal logic and quantified modal logic really are two different subjects that raise different philosophical issues and that are presently at very different stages of development. Unfortunately, however, there are very few universities that can afford the luxury of separate courses in them. Given that the subjects will be taught together, it seems a shame to make students always buy two texts. One would hope that at least some material on quantifiers would be included in subsequent editions of the book.

Like other textbooks in the field, this book stresses theoretical development of the subject at the expense of philosophical application. I think this is unfortunate. Modal logic must eventually be justified by its success in representing philosophical arguments and providing frameworks within which competing analyses of philosophical notions can be distinguished and compared. Chellas's discussion of obligation and conditionals are a step in the right direction, but I think they could have been expanded considerably. The chapter on the conditional, for example, contains no mention of counterfactual conditionals, strict implication or relevant implication. Each these topics is the subject of philosophical discussions which are interesting in their own right. Furthermore, incorporating some of this material would have made the reader appreciate better Chellas's own contributions. The uninitiated reader must wonder, for example, why Chellas is so anxious that his conditional not satisfy the rule of "strengthening the antecedent". It would also have been nice, at least in the exercises, to make students aware of applications of modal logic to topics like idealized knowledge, preference, arguments for the existence of God, imperatives, provability in arithmetic, and the verification of computer programs.

The theoretical part of the book is also somewhat limited in scope. The topics covered in detail are the traditional ones: completeness, decidability via filtrations, number of distinct modalities, alternate bases for common systems. Topics not covered include: lack of the finite model property, Bull's Theorem, correspondence results, decidability via Rabin's theorem, incompleteness, connections between modal and intuitionistic logic, general completeness proofs like Kit Fine's for extensions of K4.3 and David Lewis's for "non-iterative" logics. I am not suggesting that all of these topics be included, but perhaps some should. In particular we now have(thanks to van Benthem) incomplete logics simple enough to be presented in an introductory text. (To be fair, this material was not available at the time *Modal Logic* was written, but one would hope that something about this topic will appear in subsequent editions.) When these topics are added it will become clear that it is natural to consider *frames* from the outset as well as models.

The material that *is* included in the book is well organized, well-written and carefully edited. The only misprints of consequence that appear on the author's errata sheet are a "**" that should be a "*" on page 31, line 16, an alpha that should be a beta on the third to last line of page 41, a "definition 5.2" that should be "definition 5.5" on page 172 line 7, an "R" that should be "$R^{\#}$" on the fourth to last line of page 101. In addition to these there is a "PL" on page 21 sixth line from the bottom that should be an "RPL", and an "RN" on page 123, seventh line from the bottom that should be an "RK". The inductive definitions on pages 26 and 104 are incomplete. Exercises 5.2, 5.7 and 7.32 ask the student to find any models that verify certain sentences. Unless the instructor gives more specific directions, the clever student is likely to find easy, but uninteresting solutions. Exercise 2.39 appears to require that students find an undecidable set without having seen any previously. Finally on page 62 it is stated that the set of all finite models is effectively enumerable. An exercise which follows asks the student to show that _any_ set of finite models is effectively enumerable. On the author's errata sheet these statements are corrected to say that the set of all finite models _relevant to a sentence A_ is enumerable and that the student should show that any set of finite models _relevant to A_ is enumerable. The emendation of the text is satisfactory (provided we understand isomorphic models to be identical). The exercise, however, is still impossible. _Some_ sets of finite models are not effectively enumerable. All we really need to know, however, is that the set of all finite models is effectively enumerable and that the question of whether a finite model satisfies a schema is effectively answerable.

These few errors notwithstanding, _Modal Logic: An Introduction_ is a superior textbook. It deserves to be used widely, and I believe it will be.

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