decidability

as a property of sets, the existence of an effective procedure (a "decision procedure") which, when applied to any object, determines whether or not the object belongs to the set. A theory or logic is decidable if the set of its theorems is. Decidability is proved by describing a decision procedure and showing that it works. The TRUTH TABLE method, for example, establishes that classical propositional logic is decidable. To prove that something is not decidable requires a more precise characterization of the notion of effective procedure. Using one such characterization (for which there is ample evidence), Alonzo Church proved that classical predicate logic is not decidable.

(See also CHURCH'S THESIS, TURING MACHINE.)

deduction theorem

A result about certain systems of formal logic relating derivability and the conditional. It states that if a formula B is derivable from A (and possibly other assumptions), then the formula $A \rightarrow B$ is derivable without the assumption of A: in symbols, if $\Gamma \cup \{A\} \vdash B$ then $\Gamma \vdash A \rightarrow B$. The thought is that, for example, if *Socrates is mortal* is derivable from the assumptions *All men are mortal* and *Socrates is a man* then *If Socrates is a man he is mortal* is derivable from *All men are mortal*. Likewise, *If all men are mortal then Socrates is mortal* is derivable from *Socrates is a man*. In general, the deduction theorem is a significant result only for axiomatic or Hilbert-style formulations of logic. In most natural deduction formulations a rule of CONDITIONAL PROOF explicitly licenses derivations of $A \rightarrow B$ from Γ when B has been derived from $\Gamma \cup \{A\}$, and so there is nothing to prove.

Formal Language

A language in which an expression's grammaticality and interpretation (if any) are determined by precisely defined rules that appeal only to the form or shape of the symbols that comprise it (rather than, for example, to the intention of the speaker). It is usually understood that the rules are finite and effective (so that there is an algorithm for determining whether an expression is a formula) and that the grammatical expressions are *uniquely readable*, i.e., they are generated by the rules in only one way. A paradigm example is the language of first order PREDICATE LOGIC, deriving principally from the *Begriffschrift* of Gottlob FREGE. The grammatical formulas of this language can delineated by an inductive definition: (1) A capital letter 'F', 'G', or 'H', with or without a numerical subscript, followed by a string of lower case letters 'a','b', or 'c', with or without numerical subscripts, is a formula; (2) If A is a formula, so is ¬A; (3)If A and B are formulas, so are (A∧B), (A∨B), and (A→B). (4)If A is a formula and v is a lower case letter 'x', 'y', or 'z', with or without numerical subscripts, then ∀vA′ and ∃vA′, are formulas where A′ is obtained by replacing one or more occurrences of some lower case letter in A (together with its subscripts if any) by v; (5)nothing is a formula unless it can be shown to be one by finitely many applications of the clauses 1-4. This definition uses the device of metalinguistic variables: clauses with 'A' and 'B' are to be regarded as abbreviations of all the clauses that would result by replacing these letters uniformly by names of expressions. It also uses several naming conventions: a string of symbols is named by enclosing it within single quotes and also by replacing each symbol in the string by its name; the symbols '→', '(',')','∧','∨','¬' are considered names of themselves. The interpretation of predicate logic is spelled out by a similar inductive definition of truth in a model. With appropriate conventions and stipulations, alternative definitions of formulas can be given that make expressions like '(P→Q)' the names of formulas rather than formulas themselves. On this approach, formulas need not be written symbols at all and form cannot be identified with shape in any narrow sense. For TARSKI, CARNAP and others, a formal language also included rules of "transformation" specifying when one expression can be regarded as a consequence of others. Today it is more common to view the language and its consequence relation as distinct. Formal languages are often contrasted with natural languages, like English or Swahili. Richard MONTAGUE, however, has tried to show that English is itself a formal language, whose rules of grammar and interpretation are similar to--though much more complex than--predicate logic.

Formalism

The view that mathematics concerns manipulations of symbols according to prescribed structural rules. It is cousin to NOMINALISM, the older and more general metaphysical view that denies the existence of all abstract objects and is often contrasted with PLATONISM, which takes mathematics to be the study of a special class of non-linguistic, non-mental objects, and INTUITIONISM, which takes it to be the study of certain mental constructions. In sophisticated versions, mathematical activity can comprise the study of possible formal manipulations within a system as well as the manipulations themselves and the "symbols" need not be regarded as either linguistic or concrete. Formalism is often associated with the famous mathematician David HILBERT. But Hilbert held that the "finitary" part of mathematics including, for example, simple truths of arithmetic, described indubitable facts about real objects and that the "ideal" objects that feature elsewhere in mathematics are introduced to facilitate research about the real objects. Hilbert's formalism is the view that the *foundations* of mathematics can be secured by proving the consistency of formal systems to which mathematical theories are reduced. Gödel's two INCOMPLETENESS THEOREMS establish important limitations on the success of such a project.

Formalize

Narrowly construed, to formulate a subject as a theory in first order PREDICATE LOGIC; broadly construed, to describe the essentials of the subject in some FORMAL LANGUAGE for which a notion of consequence is defined. For HILBERT, formalizing mathematics requires at least that there be finite means of checking purported proofs. See FORMALIZATION, PROOF THEORY.

ideal language

A system of notation that would correct perceived deficiencies of ordinary language by requiring the structure of expressions to mirror the structure of that which they represent. The notion that conceptual errors can be corrected and philosophical problems solved (or dissolved) by properly representing them in some such system figured prominently in the writings of LEIBNIZ, CARNAP, RUSSELL, WITTGENSTEIN, and FREGE, among others. For Russell, the ideal, or "logically perfect," language is one in which grammatical form coincides with LOGICAL FORM, there are no vague or ambiguous expressions, and no proper names that fail to denote. Frege's *Begriffschrift* is perhaps the most thorough and successful execution of the ideal language project. A wide range of deductive arguments (notably those from mathematics) can be represented within this system or its modern descendants and effectively checked for correctness. (See FORMAL LANGUAGE.)

is

third person singular form of the verb *to be*. In various forms and senses, the word pervades our language, so that providing an adequate characterization without using it poses a certain challenge. (I will try to get by with mention, but no use.) Philosophers distinguish among at least three fundamental senses of *is*, according to the resources required for a proper logical representation. The *is* of existence (*There is a unicorn in the garden* / $\exists x(Ux \wedge Gx)$) uses the existential quantifier. The *is* of identity (*Hesperus is Phosphorus* / $j=k$) employs the predicate of identity. The *is* of predication (*Sampson is strong* / *Sj*) merely juxtaposes predicate symbol and proper name. Some controversy attends the first sense. Some (notably MEINONG) maintain that *is* applies more broadly than *exists*, the former producing true sentences when combined with *deer* and *unicorn* and the latter producing true sentences when combined with *deer* but not *unicorn*. Others (like AQUINAS) take *being* (*esse*) to denote some special activity that every existing object necessarily performs, which would seem to imply that with *is* they attribute more to an object than we do with *exists*. Other issues arise in connection with the second sense. Does *Hesperus is Phosphorus*, for example, attribute anything more to the heavenly body than its identity with itself? Consideration of such a question led FREGE to conclude that names (and other meaningful expressions) of ordinary language have a "sense" or "mode of presenting" the object to which they refer that representations within our standard, extensional logical systems fail to expose. The distinction between the *is* of identity and the *is* of predication parallels Frege's distinction between object and concept: words signifying objects stand to the right of the *is* of identity those signifying concepts stand to the right of the *is* of predication. Although it seems remarkable that so many deep and difficult philosophical concepts should link to a single short and commonplace word, we should perhaps not read too much into that observation. Some languages divide the various roles played by English's compact copula among several constructions, and others use the corresponding word for other purposes.

See also EXISTENTIAL IMPORT, IDENTITY.

Kripke semantics

A type of formal semantics for languages with operators □ and ◊ for NECESSITY and possibility. (The labels *possible worlds semantics* and *relational semantics* are sometimes used for the same notion.) Also a similar semantics for INTUITIONISTIC LOGIC. In a basic version a *frame* for a sentential language with □ and ◊ is a pair (W,R) where W is a non-empty set (the "POSSIBLE WORLDS") and R is a binary relation on W (the relation of "relative possibility" or "accessibility"). A *model* on the frame (W,R) is a triple (W,R,V), where V is a function (the "valuation function") that assigns truth values to sentence letters at worlds. If wεW then a sentence □**A** is *true at world w in the model (W,R,V)* if **A** is true at all worlds vεW for which wRv. (Informally, □**A** is true at w if **A** is true at all the worlds that would be possible if w were actual. This is a generalization of the doctrine commonly attributed to Leibniz that necessity is truth in all possible worlds.) **A** is valid in the model (W,R,V) if it is true at all worlds wεW in that model. It is valid in the frame (W,R) if it is valid in all models on that frame. It is valid if it is valid in all frames. In predicate logic versions, a frame may include another component D, that assigns a non-empty set D_w of objects (*the existents at w*) to each possible world w. Terms and quantifiers may be treated either as *objectual* (denoting and ranging over individuals) or *conceptual* (denoting and ranging over functions from possible worlds to individuals) and either as *actualist* or *possibilist* (denoting and ranging over either existents or possible existents). On some of these treatments there may arise further choices about whether and how truth-values should be assigned to sentences that assert relations among non-existents.

The development of Kripke semantics marks a watershed in the modern study of modal systems. In the nineteen thirties, forties and fifties a number of axiomatizations for necessity and possibility were proposed and investigated. Rudolf CARNAP showed that, for the simplest of these systems, C.I. LEWIS's S5, □**A** can be interpreted as saying that **A** is true in all "state descriptions". Answering even the most basic questions about the other systems, however, required effort and ingenuity. In the late fifties and early sixties Stig Kanger, Richard Montague, Saul Kripke, and Jaakko Hintikka each formulated interpretations for such systems that generalized Carnap's semantics by using something like the accessibility relation described above. Kripke's semantics was somewhat simpler than the others in that alternativeness was taken to be a relation among mathematically primitive "possible worlds", and, in a series of papers, Kripke showed that it could be easily modified to provide interpretations for a great variety of modal systems. For these reasons Kripke's formulation has become standard. Relational semantics provided simple solutions to some older problems about the distinctness and relative strength of the various systems. It also opened new areas of investigation, facilitating *general* results (establishing decidability and other properties for infinite classes of modal systems), *incompleteness* results (exhibiting systems not determined by any class of frames) and *correspondence* results (showing that the frames verifying certain modal formulae were exactly the frames meeting certain conditions on R). It suggested parallel interpretations for notions whose patterns of inference were known to be similar to that of necessity and possibility, including obligation and permission, epistemic necessity and possibility, provability and consistency and, more recently, the notion of a computation's inevitably or possibly terminating in a particular state. It inspired similar semantics for non-classical conditionals and the more general neighborhood or functional variety of possible worlds semantics.

The philosophical utility of Kripke semantics is more difficult to assess. Since the alternativeness relation is often explained in terms of the modal operators, it is difficult to maintain that the semantics provides an explicit analysis of the modalities it interprets. Furthermore, questions about which version of the semantics is correct (particularly for quantified modal systems) are themselves tied to substantive questions about the nature of things and worlds. The semantics does impose important constraints on the meaning of modalities and there is no doubt that it has enabled many philosophical questions to be posed more clearly and starkly.

Sortal predicate

"Person", "green apple", "regular hexagon" and "pile of coal" would generally be regarded as sortal predicates, whereas "tall", "green thing", and "coal" would generally be regarded as non-sortal predicates. An explicit and precise definition of the distinction is hard to come by. Very roughly a sortal predicate says what kind of an object something is and implies or presupposes conditions for objects of that kind to be identical. Sortal predicates are sometimes said to be characterized by the fact that they provide a criterion of counting or that they do not apply to the parts of the objects to which they apply, but there are difficulties with each of these characterizations.

The notion figures in recent philosophical discussions on a variety of topics. Robert Ackermann and others have suggested that any scientific law confirmable by observation might require the use of sortal predicates. Thus "all non-black things are not ravens", while logically equivalent to the putative scientific law "all ravens are black", is not itself confirmable by observation because "non-black" is not a sortal predicate. David Wiggins and others have discussed the idea that all identity claims are sortal-relative in the sense that an appropriate response to the claim a=b is always "the same **what** as b?". John Wallace has argued that there would be advantages in relativizing the quantifiers of predicate logic to sortals. All men are mortal would be rendered $\forall x[m]Dx$, rather than $\forall x(Mx \rightarrow Dx)$. Crispin Wright has suggested that the view that *natural number* is a sortal concept is central to Frege's (or any other) number-theoretic platonism.

The word "sortal" as a technical term in philosophy first occurs in John LOCKE's *Essay Concerning Human Understanding*. Locke argues that the so-called essence of a genus or sort (unlike the real essence of a thing) is merely the abstract idea that the general or sortal name stands for. But "sortal" has only one occurrence in Locke's *Essay*. The currency of the term in the contemporary philosophical idiom probably should be credited to P.F. Strawson's *Individuals*. The general idea may be traced back at least to the notion of "second substance" in Aristotle's *Categories*.

Truth value semantics

Interpretations of formal systems in which the truth value of a formula rests ultimately only on truth values that are assigned to its atomic subformulas (where "subformula" is suitably defined). The label is due Hugues LeBlanc. On a truth-value interpretation for first order PREDICATE LOGIC, for example, the formula atomic $\forall$**xFx** is true in a model if and only if all its *instances* **Fm**, **Fn**,... are true, where the truth value of these formulas is simply assigned by the model. On the standard Tarskian or objectual interpretation, by contrast, $\forall$**xFx** is true in a model if and only if every object in the domain of the model is an element of the set that interprets **F** in the model. Thus a truth value semantics for predicate logic comprises a SUBSTITUTIONAL interpretation of the quantifiers and a "non-denotational" interpretation of terms and predicates. If $\mathbf{t_1}$,$\mathbf{t_2}$,... are all the terms of some first order language, then there are objectual models that satisfy the set {$\exists$**x-Fx**,$\mathbf{Ft_1}$,$\mathbf{Ft_2}$,...}, but no truth value interpretations that do. One can ensure that truth-value semantics delivers the standard logic, however, by suitable modifications in the definitions of consistency and consequence. A set **Γ** of formulas of language L is said to be consistent, for example, if there is some **Γ'** obtained from **Γ** by relettering terms such that **Γ'** is satisfied by some truth-value assignment, or, alternatively, if there is some language L^+ obtained by adding terms to L such that **Γ** is satisfied by some truth-value assignment to the atoms of L^+.

Truth value semantics is of both technical and philosophical interest. Technically, it allows the completeness of first order predicate logic and a variety of other formal systems to be obtained in a natural way from that of PROPOSITIONAL LOGIC. Philosophically, it dramatizes the fact that the formulas in one's theories about the world do not, in themselves, determine one's ontological commitments. It is at least possible to interpret first order formulas without reference to special domains of objects, and higher order formulas without reference to special domains of relations and properties.

The idea of truth value semantics dates at least to writings of E.W. Beth on first order predicate logic in 1959 and of K. Schütte on simple TYPE THEORY in 1960. In more recent years similar semantics have been suggested for SECOND ORDER LOGICS, MODAL and TENSE LOGICS, INTUITIONISTIC LOGIC and SET THEORY.